

**Master 2 de Dynamique des Fluides et Energétique
2020-2021**

Turbulence

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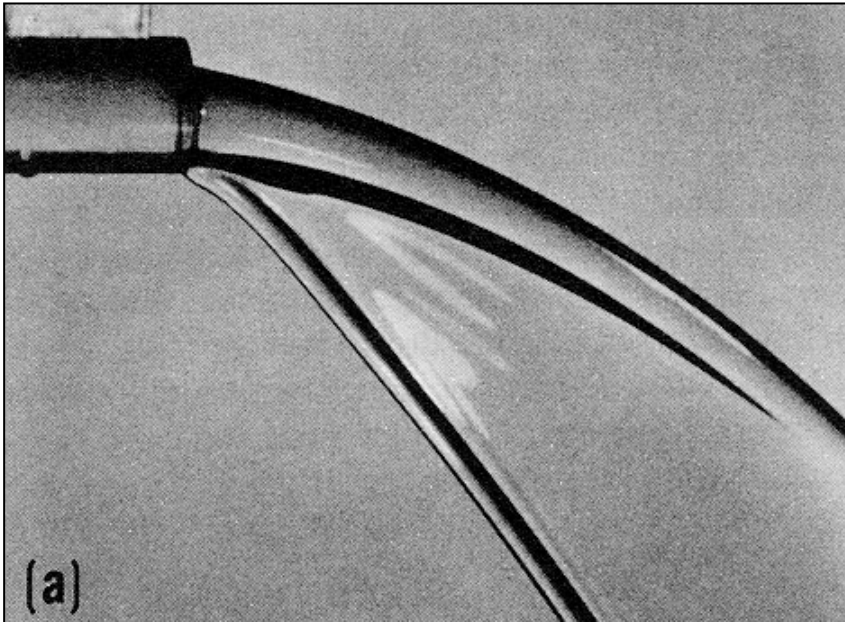
Plan du cours

1. Equations de Reynolds (RANS)
 1. Rappels cinématiques, équations fondamentales
 2. Décomposition de Reynolds, tenseur de Reynolds
 3. Limitations
2. Modélisation de turbulence
 1. Modèles de “viscosité turbulente”
 1. Définition, phénoménologie
 2. Modèles à “0 équation” : Longueur de mélange
 3. Modèles à “2 équations” : **k- ϵ** , k- ω
 2. Modèles de contraintes de Reynolds (R_{ij} - ϵ)
 3. Large eddy simulation (LES)
3. Turbulence homogène isotrope
 1. Théorie de Kolmogorov
 2. Description spectrale de la turbulence homogène
 3. Fonctions de structures, lois d’échelles
 4. Intermittence ; développements récents
4. Codes industriels, LES [S. Deck, ONERA]

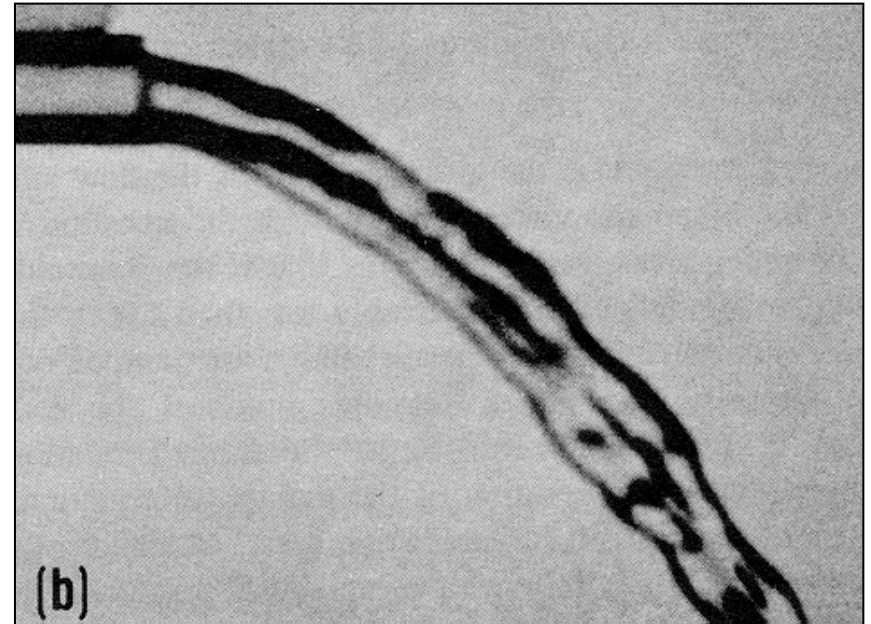
La turbulence “frotte plus”

Augmentation de la perte de charge due au melange de quantité de mouvement

$Re < Re_c$

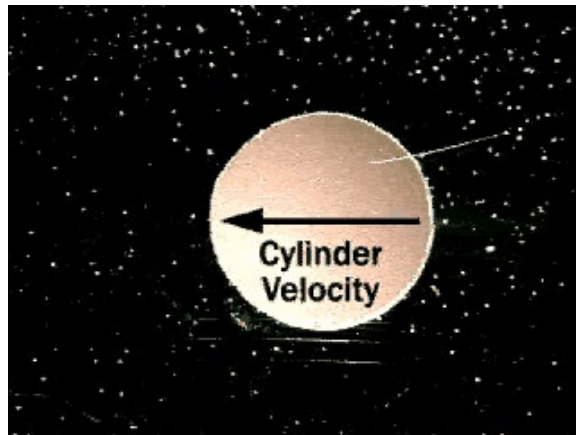


$Re > Re_c$



Transition laminaire - turbulent

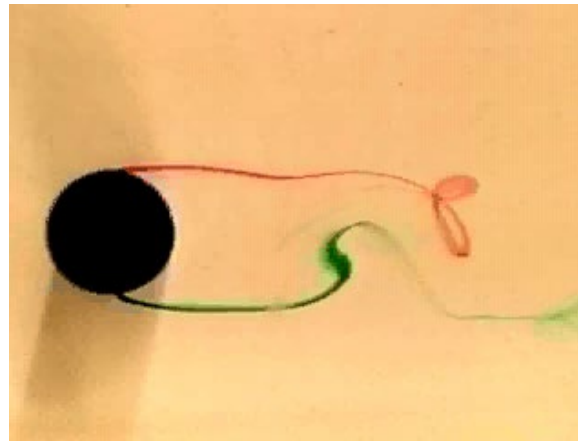
Inertie nulle



"laminaire"
($Re \ll 1$)

$$F \sim v$$

Inertie modérée



"instable"
($Re \sim 1$)
(10-1000...)

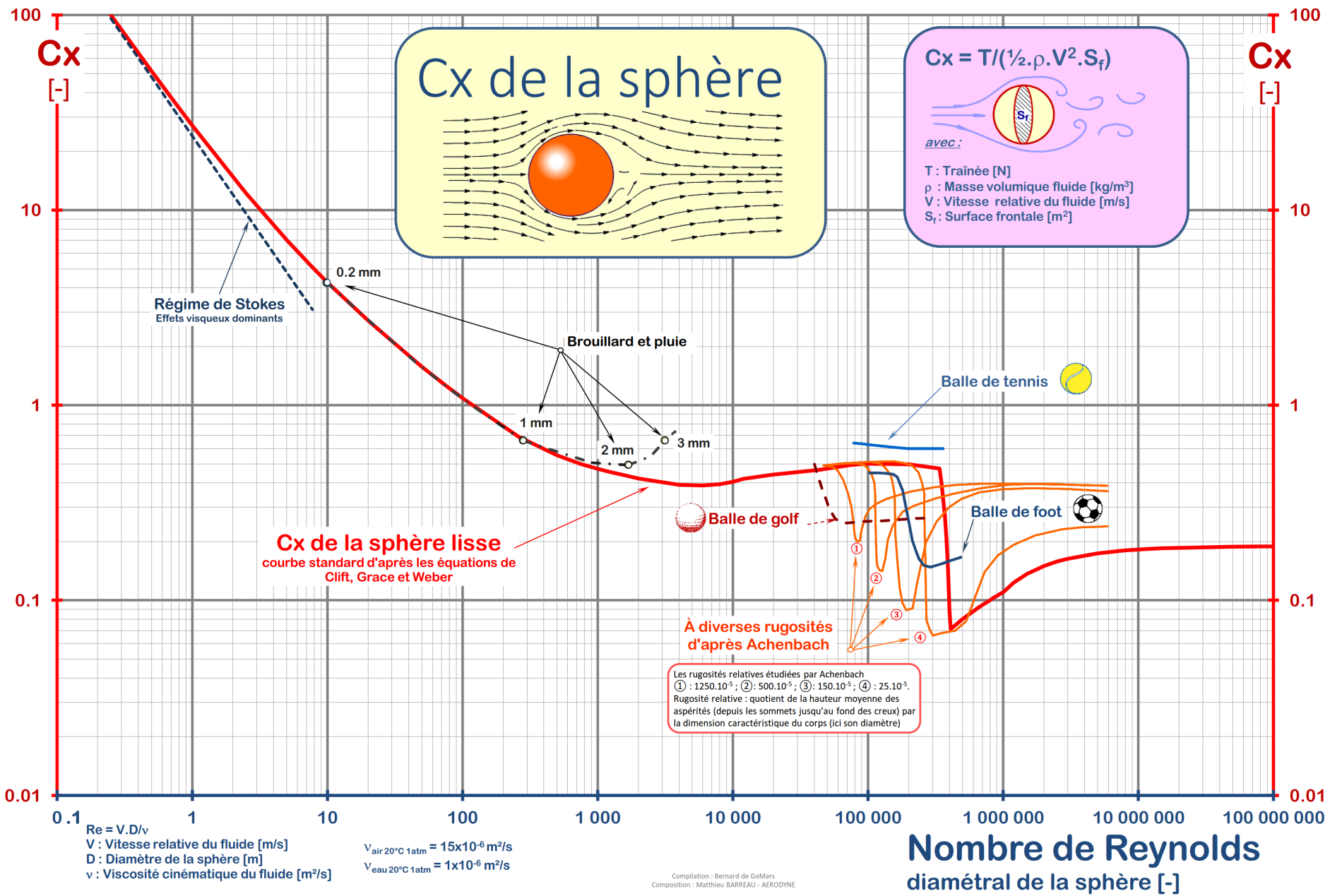
Inertie forte



"turbulent"
($Re \gg 1$)

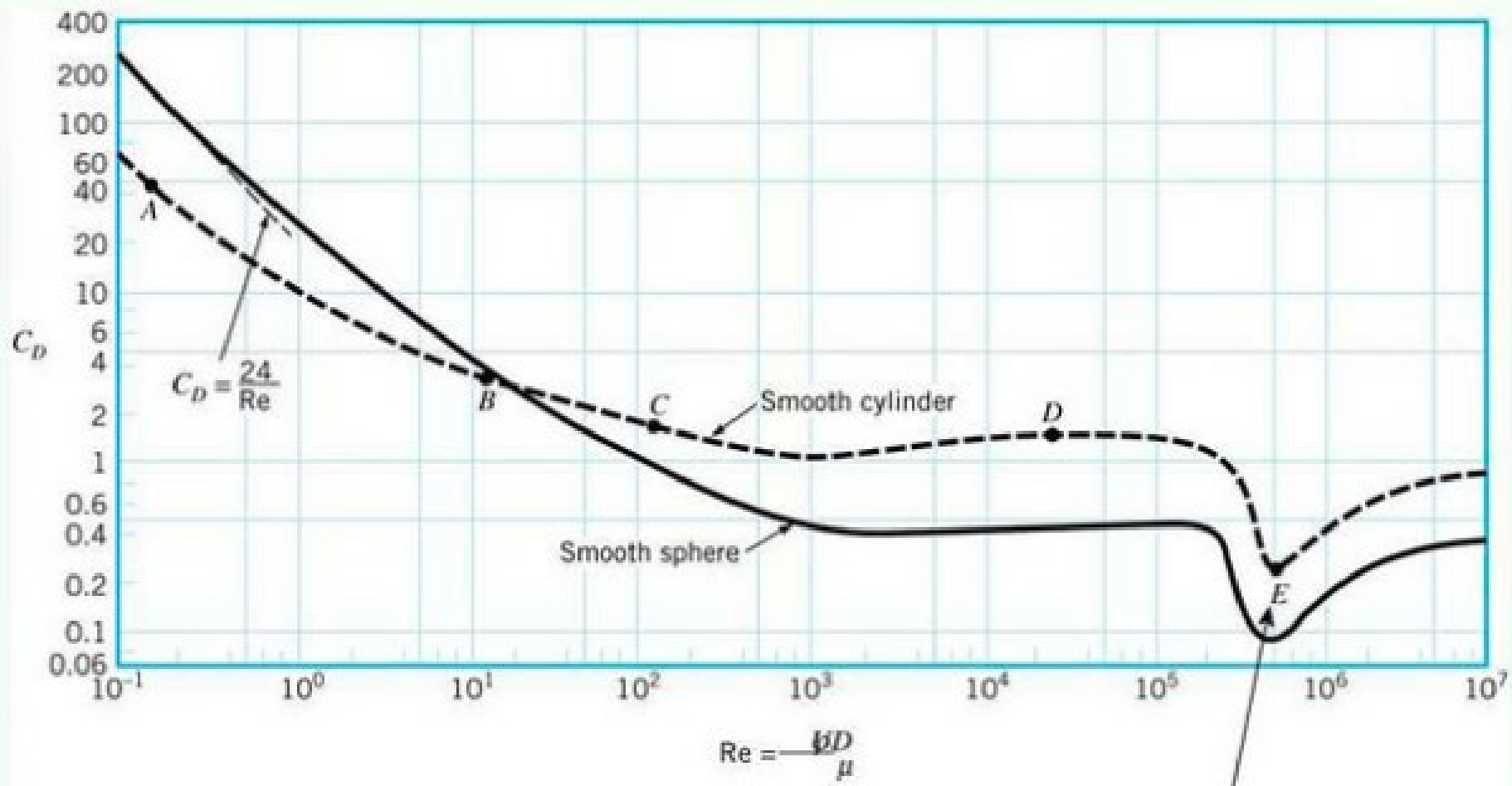
$$F \sim v^2$$

"frotte plus"

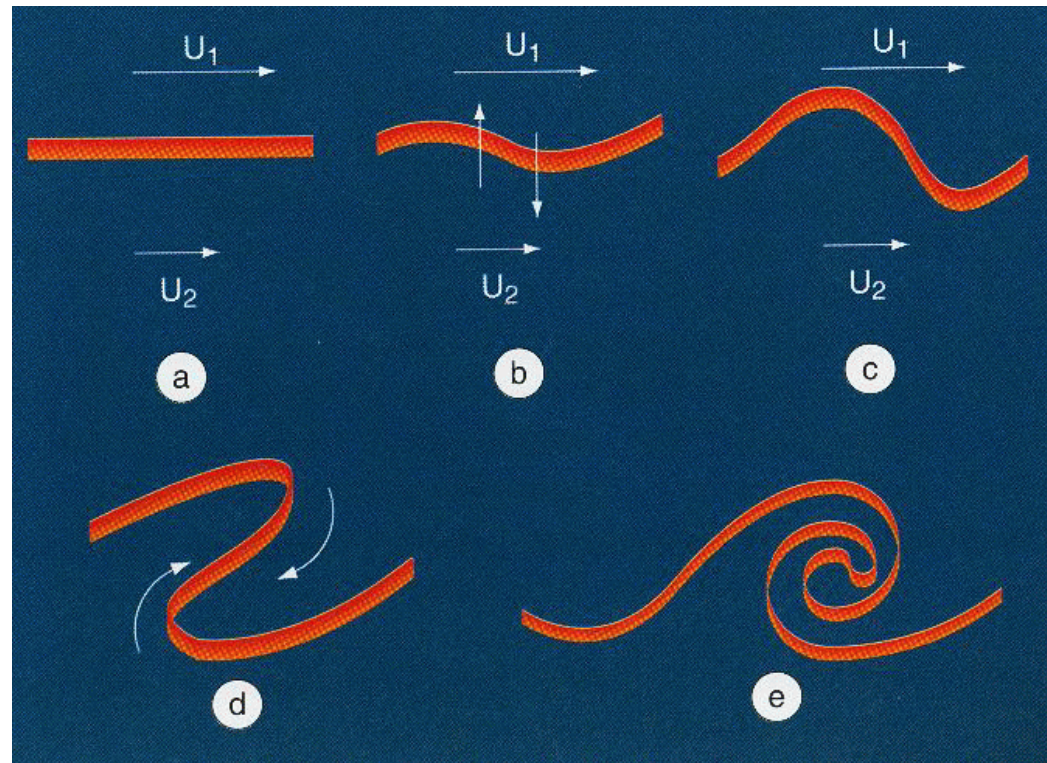
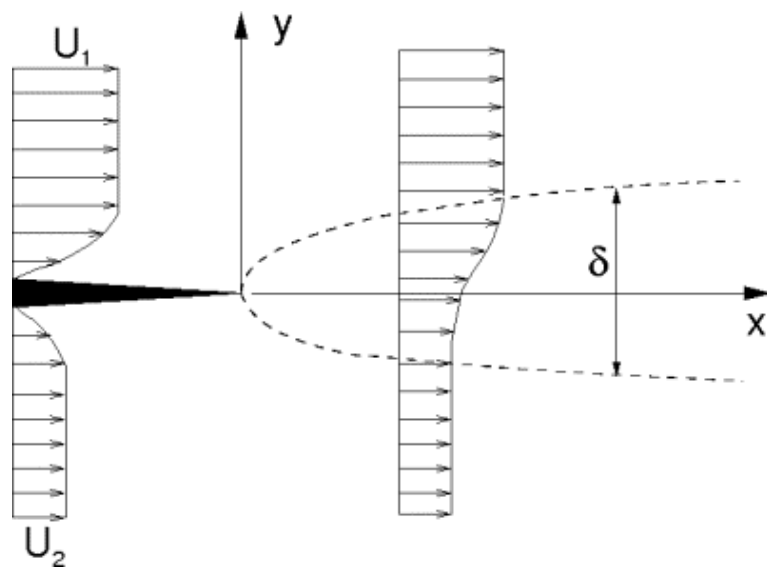


La crise de traînée

Coefficient of Drag as a function of Reynolds Number



Ecoulements de cisaillement

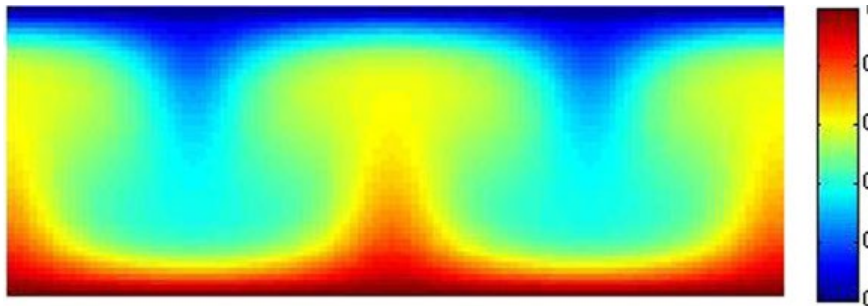
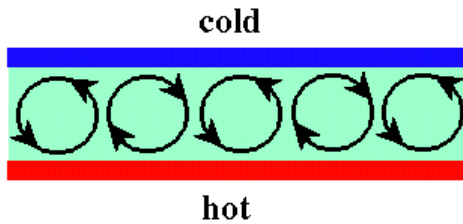


Shear layer: Kelvin Helmholtz instability

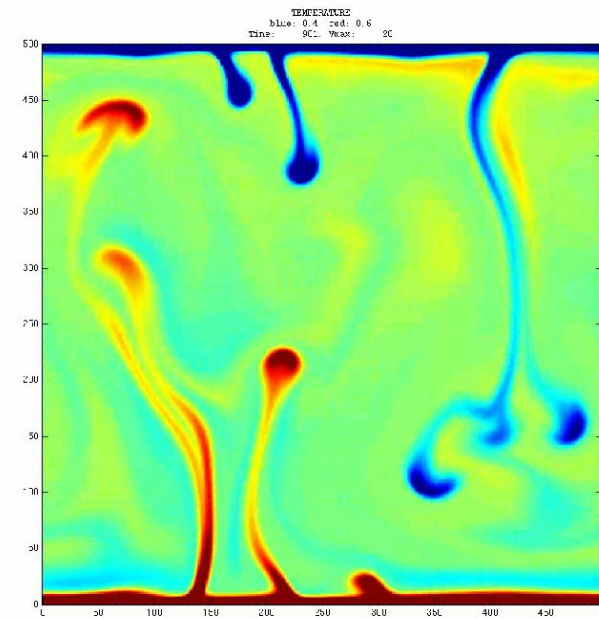




Convection de Rayleigh-Bénard

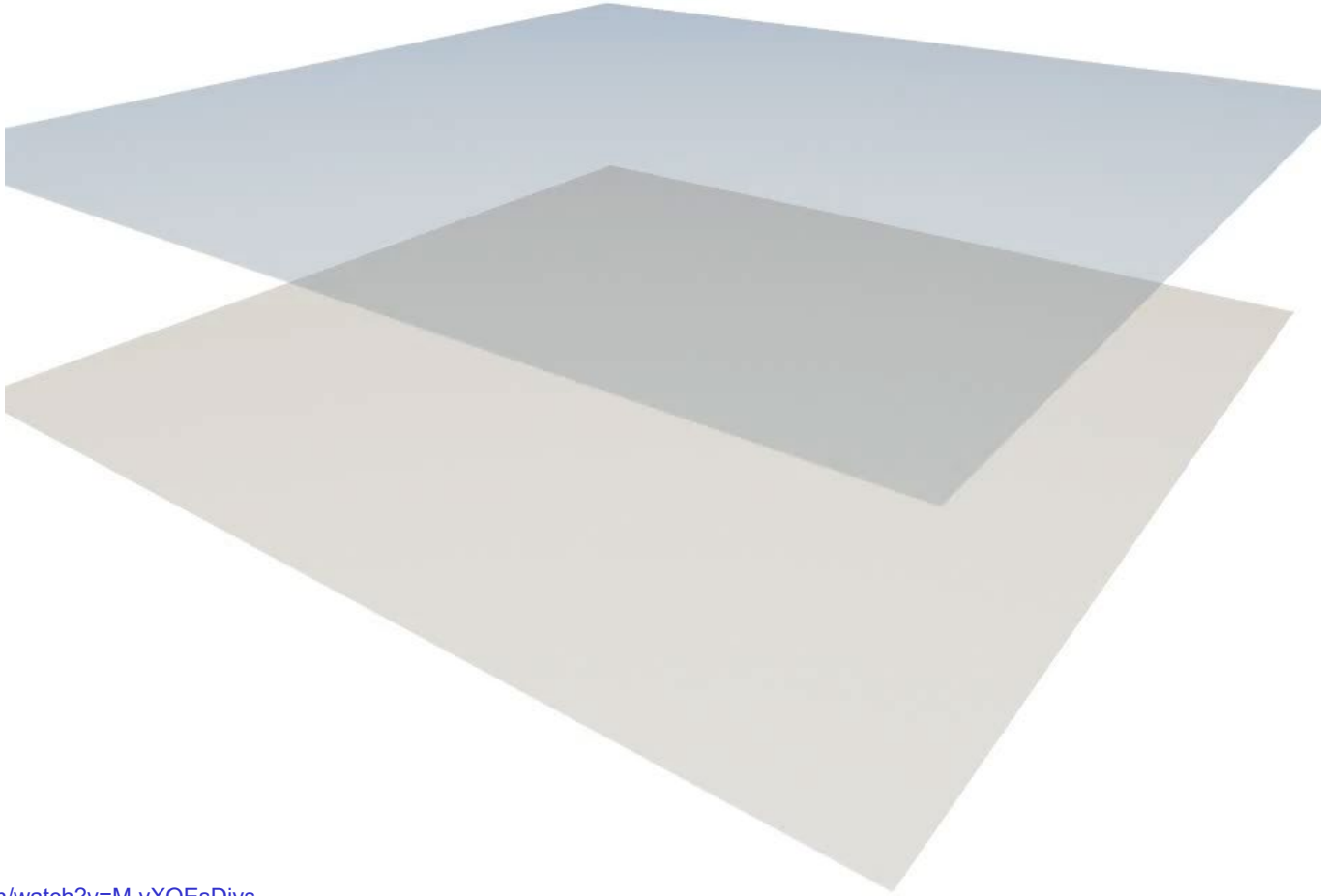


$Ra > Ra_c$



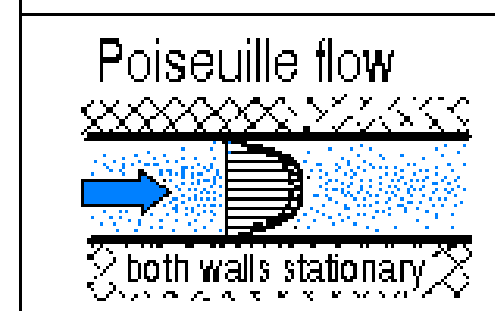
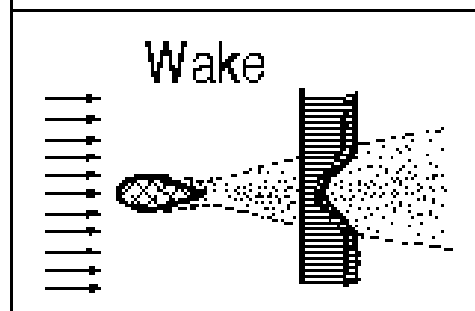
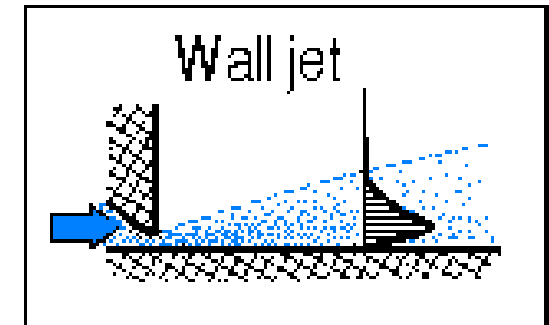
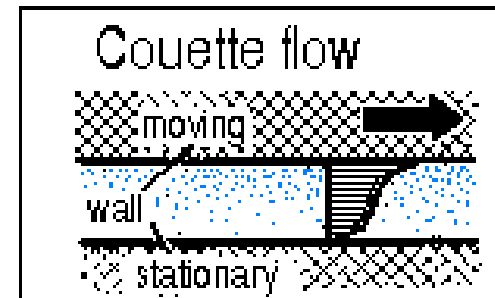
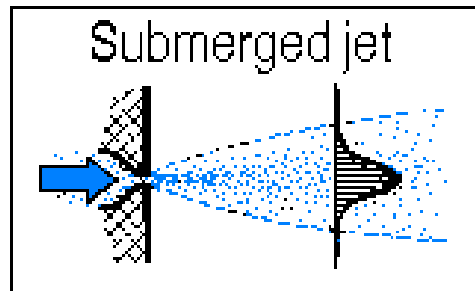
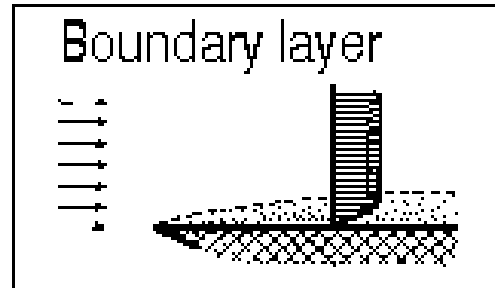
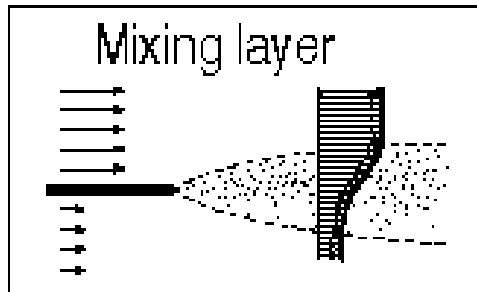
$Ra \gg Ra_c$

Convection de Rayleigh-Bénard

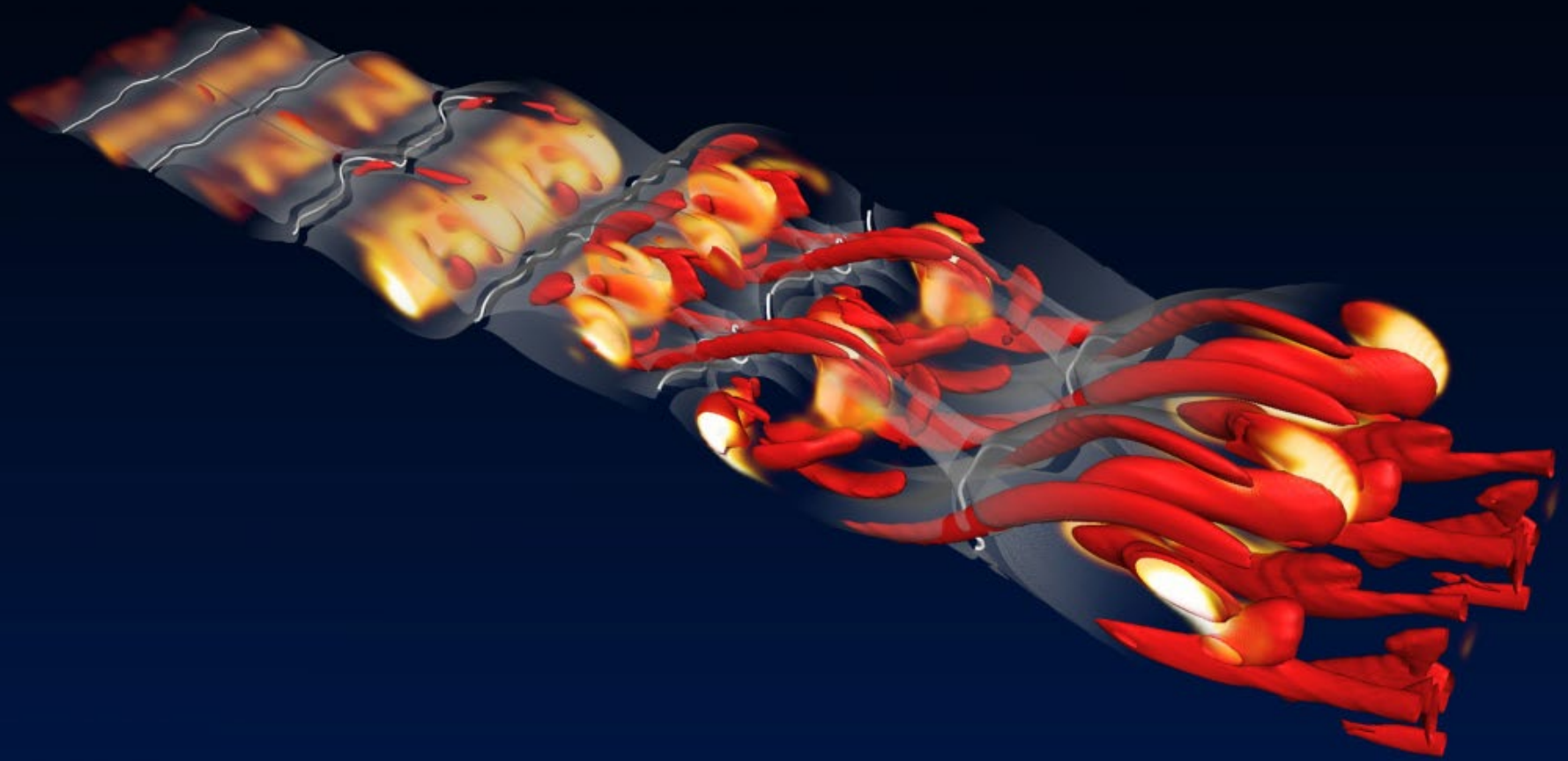


<https://www.youtube.com/watch?v=M-yXOEsdvIs>

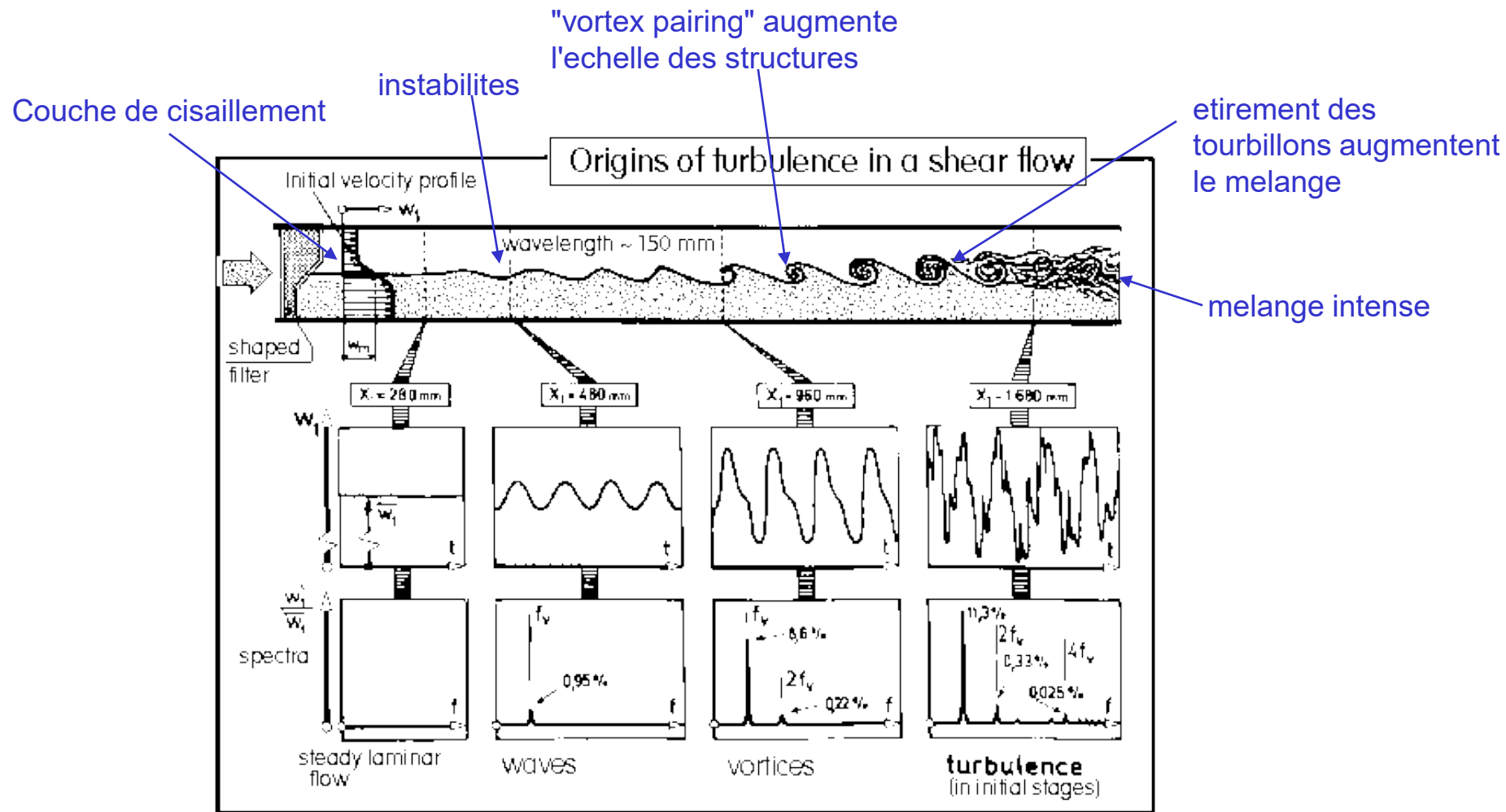
Ecoulements quasi-parallèles $u_x(y)$



Shear layer: Kelvin Helmholtz instability



Instabilités d'une couche de mélange

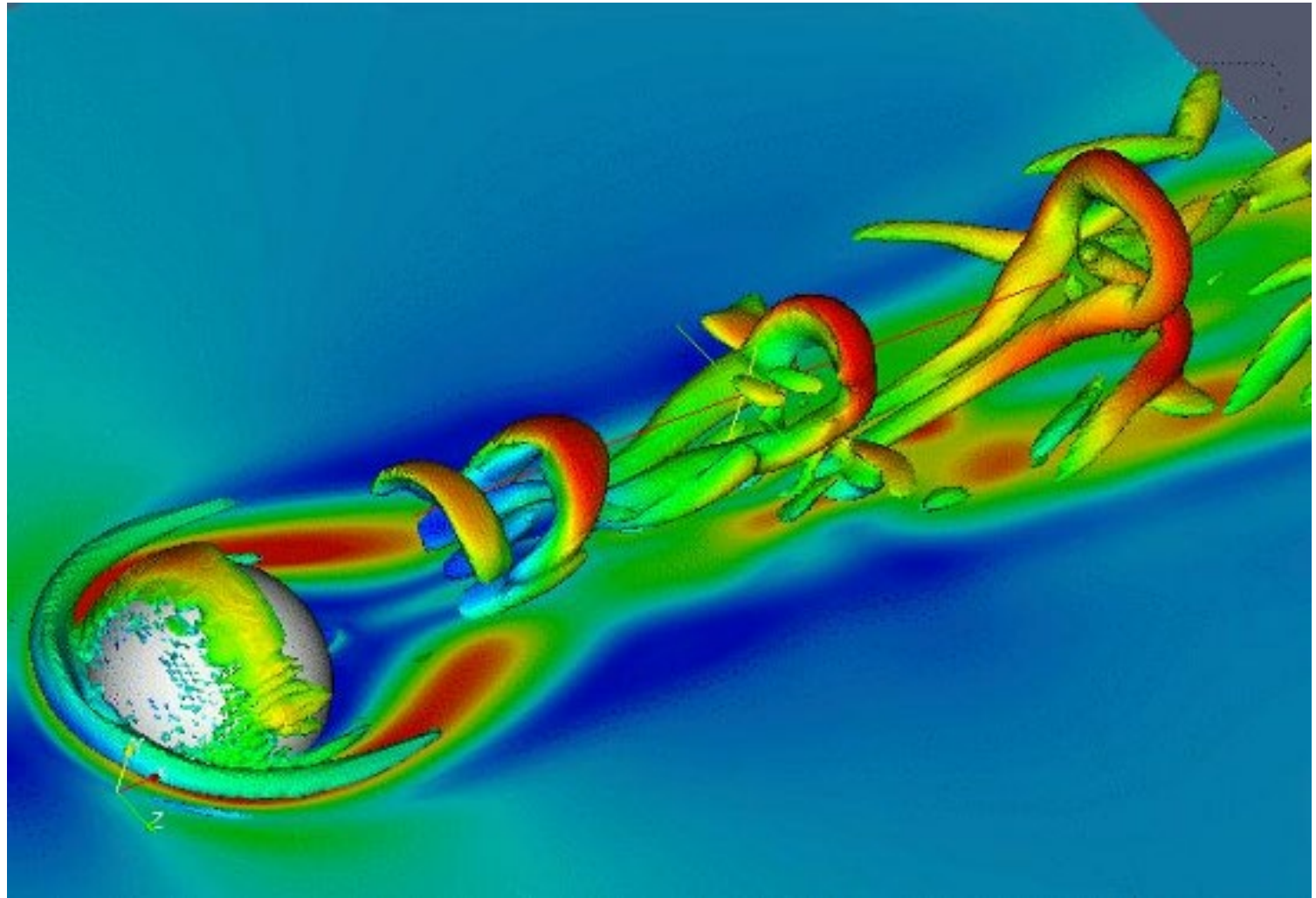


Turbulent jet (experimental)



Turbulent jet (DNS)







Re = 22 000

Turbulent boundary layer (experiment)



Turbulent boundary layer (DNS)

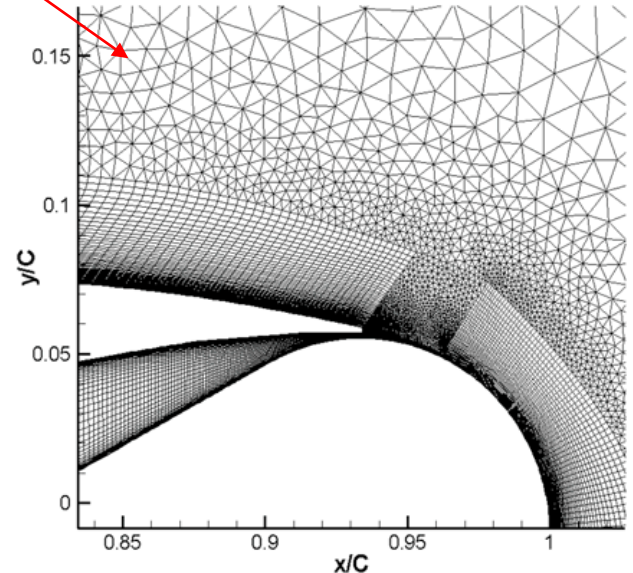
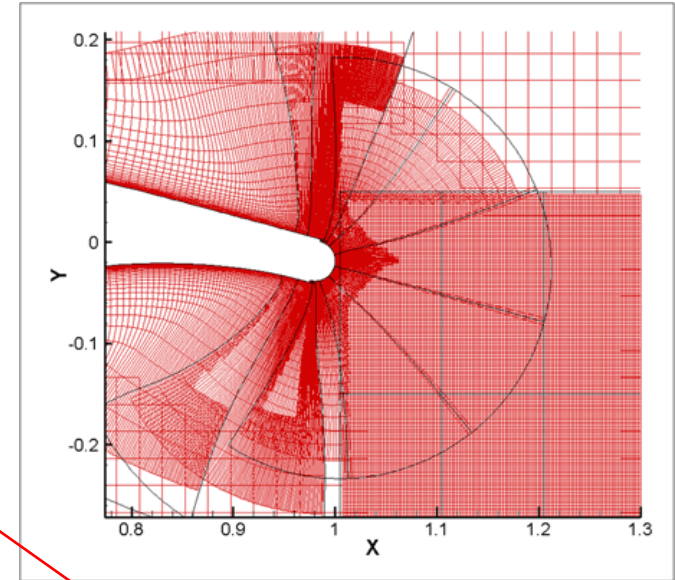


Simulation: Approche Directe (DNS)

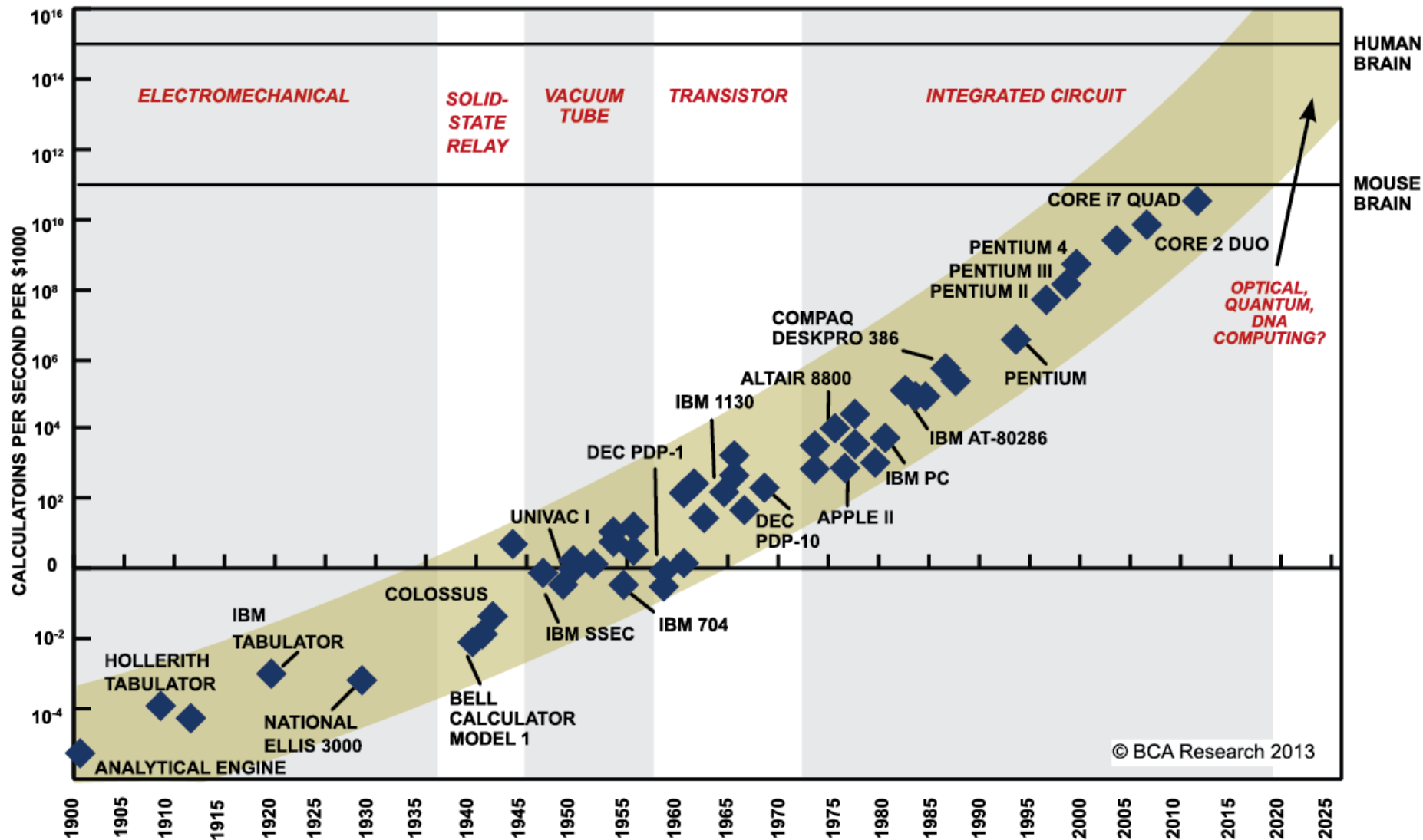
- DNS: Resolution directe des équations de N-S

$$u_x, u_y, u_z, p = f(x_i, y_i, z_i, t)$$

- Nombre de dds nécessaires:
 - calcul 3D instationnaire
 - $\Delta x \sim \eta/4$, $\Delta t \sim \tau/4$,
avec $\eta = (\nu^3/\varepsilon)^{1/4}$ et $\tau = (\nu/\varepsilon)^{1/2}$
 - memoire: $N^3 \sim (L/\Delta x)^3 = 64Re^{9/4}$
 - temps CPU $\sim N^3/\Delta t \sim Re^{11/4}$



La Loi de Moore

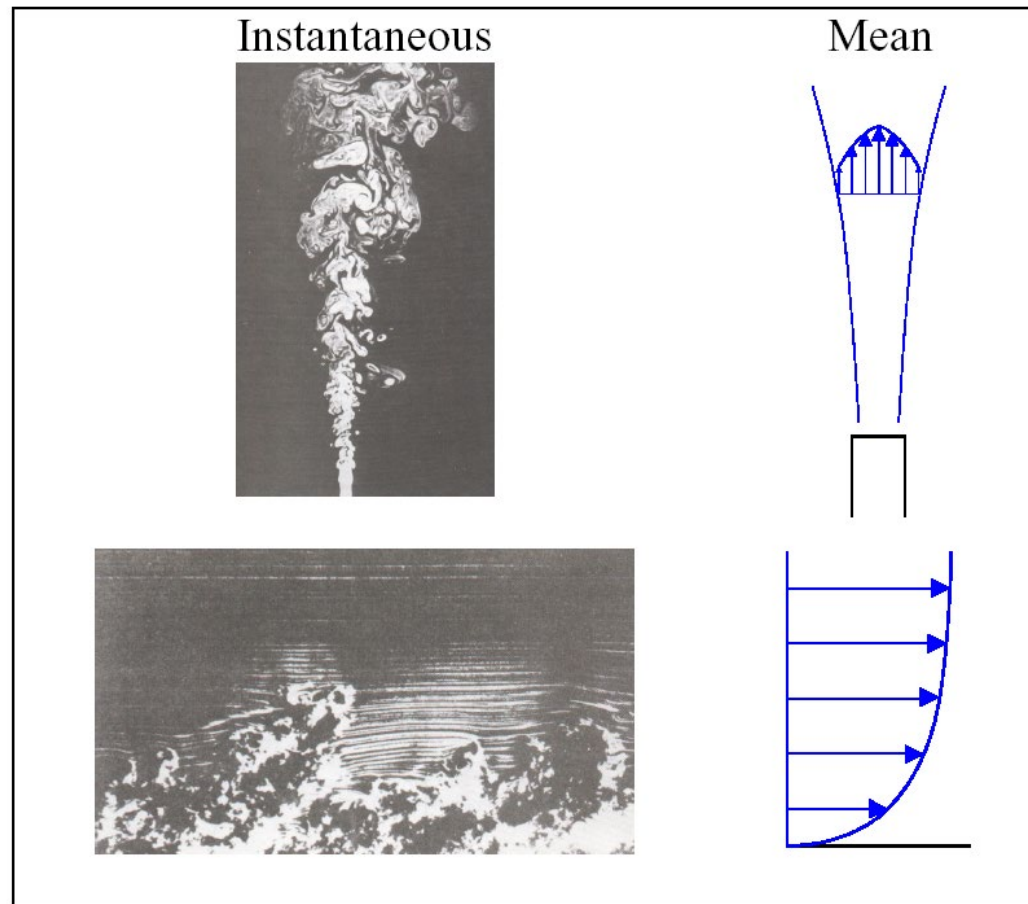


SOURCE: RAY KURZWEIL, "THE SINGULARITY IS NEAR: WHEN HUMANS TRANSCEND BIOLOGY", P.67, THE VIKING PRESS, 2006. DATAPPOINTS BETWEEN 2000 AND 2012 REPRESENT BCA ESTIMATES.

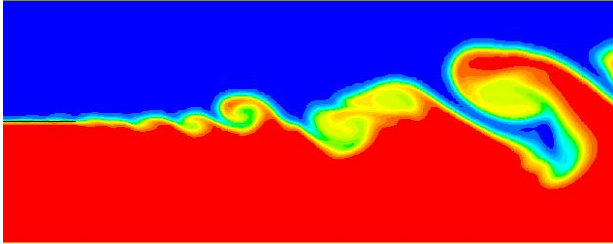
Approche statistique

Décompositions :

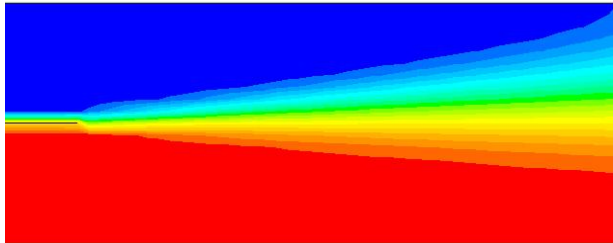
- Moyenne [d'ensemble] + fluctuations (RANS)
- Grandes échelles + petites échelles (LES)



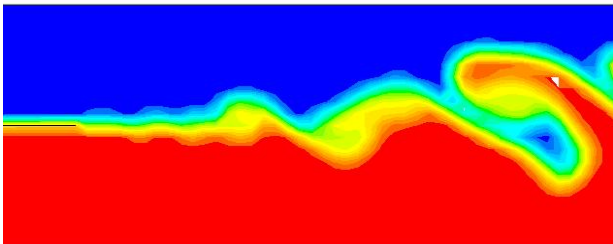
Approche statistique



DNS : Direct Numerical Simulation
3D, unsteady

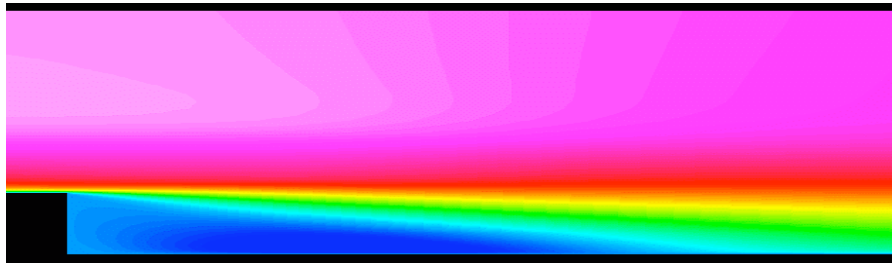
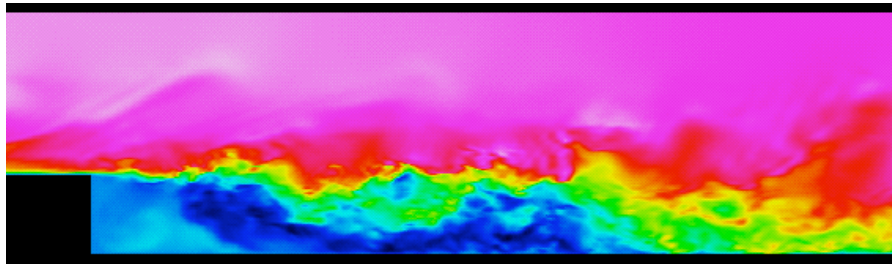


RANS : Reynolds-Averaged Navier-Stokes
Steady / unsteady



LES : Large Eddy Simulation
3D, unsteady

Marche descendante : moyenne d'ensemble



Reynolds number: 5100

Expansion ratio: 1.2

Grid: 768 x 192 x 64

***Normalization based on
step height H and
free-stream velocity at inlet***

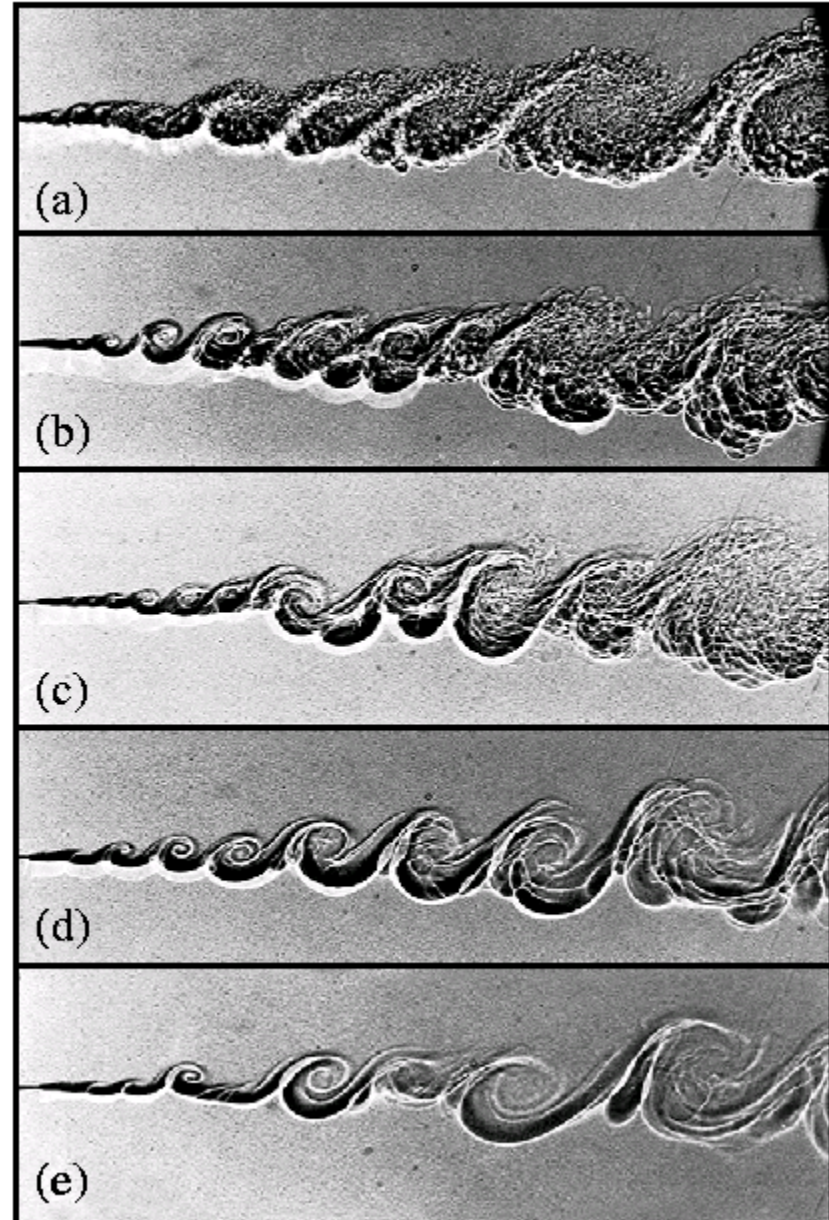
Structures cohérentes

Persistence de structures cohérentes :
- Vibrations, bruit aérodynamique

Problème pour **RANS**

Solution : **LES** ou **DNS**

Couche de mélange plane
à différents nombres de Reynolds
(a) haut Re
(e) bas Re



Codes CFD (computational fluid dynamics)

Codes industriels courants :

Ansys (Fluent, Gambit, CFX,...)

CD-adapco (Star-CD)

FlowScience (Flow3D)



Viscous Model

Model

- ☐ Inviscid
- ☐ Laminar
- ☐ Spalart-Allmaras [1 eqn]
- ☒ k-epsilon [2 eqn]
- ☐ k-omega [2 eqn]
- ☐ Reynolds Stress [5 eqn]

k-epsilon Model

- ☒ Standard
- ☐ RNG
- ☐ Realizable

Near-Wall Treatment

- ☒ Standard Wall Functions
- ☐ Non-Equilibrium Wall Functions
- ☐ Enhanced Wall Treatment
- ☐ User-Defined Wall Functions

Options

☐ Viscous Heating

Model Constants

Cmu: 0.09

C1-Epsilon: 1.44

C2-Epsilon: 1.92

TKE Prandtl Number: 1

User-Defined Functions

Turbulent Viscosity: none

Prandtl Numbers

TKE Prandtl Number: none

TDR Prandtl Number: none

Energy Prandtl Number: none

OK Cancel Help

(Fluent)

Modèles hybrides RANS-LES



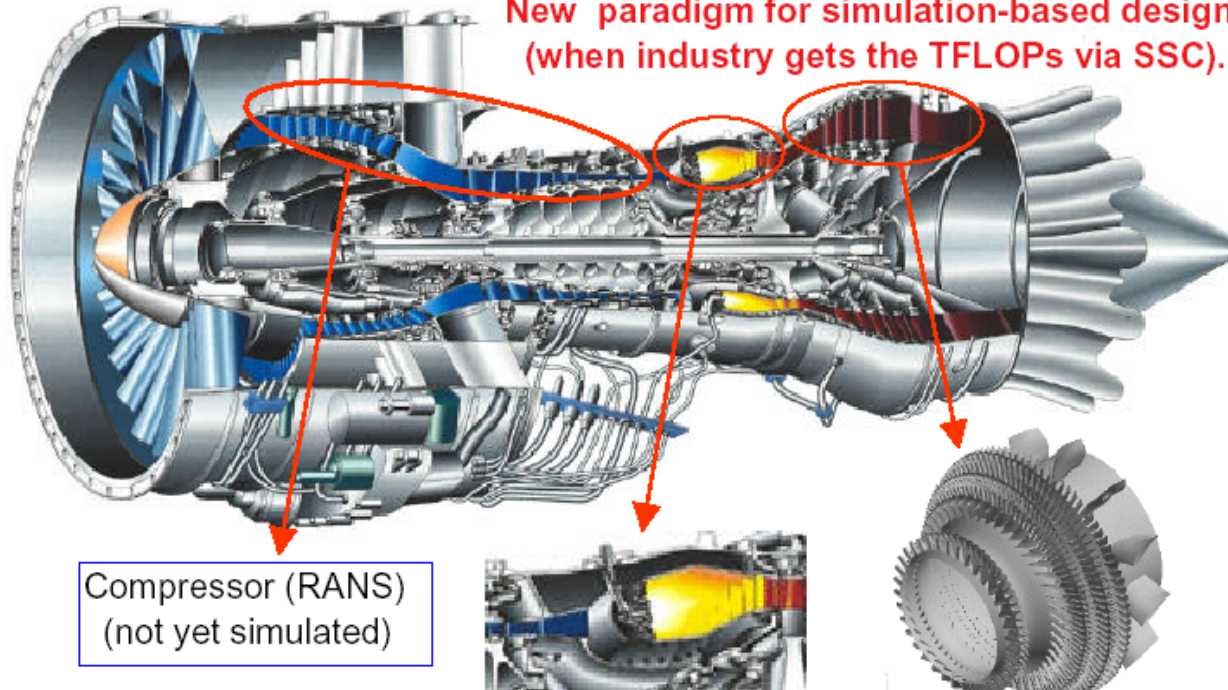
CITS simulation: PW6000 Engine



Goal: Integrated flow and combustion simulation

Study combustion instabilities, blade vibration,...

New paradigm for simulation-based design
(when industry gets the TFLOPs via SSC).



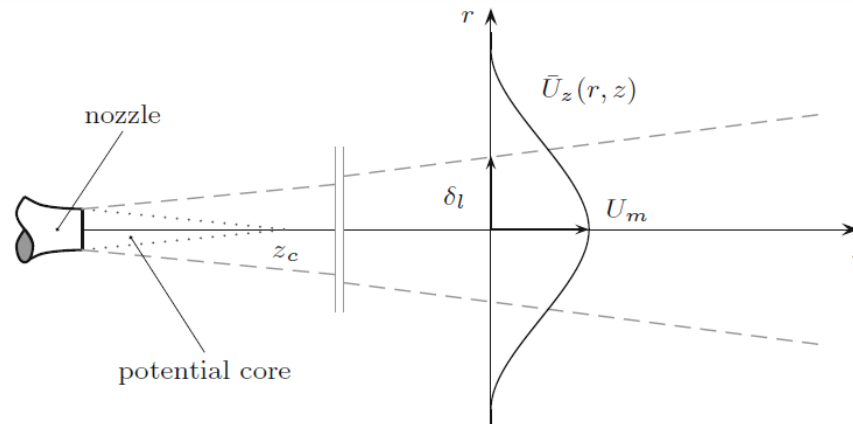
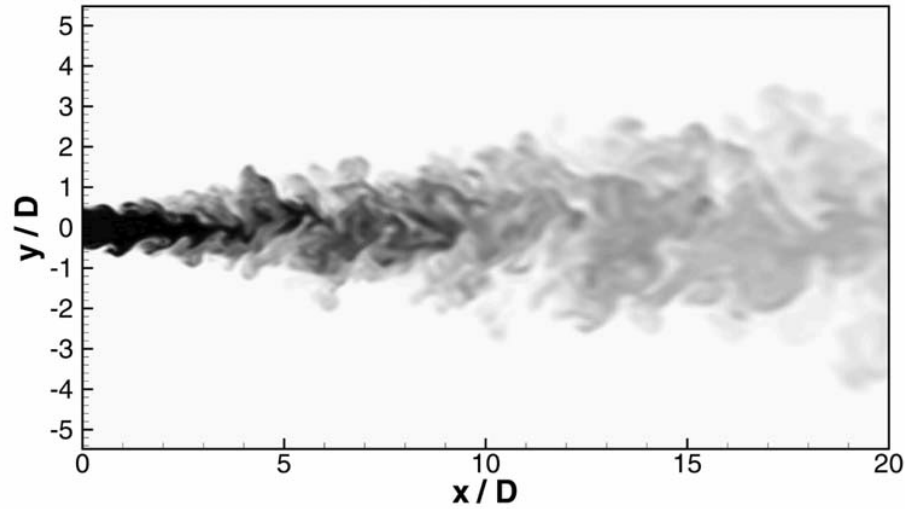
Compressor (RANS)
(not yet simulated)

Combustor (LES)
(RANS by NASA's NCC)

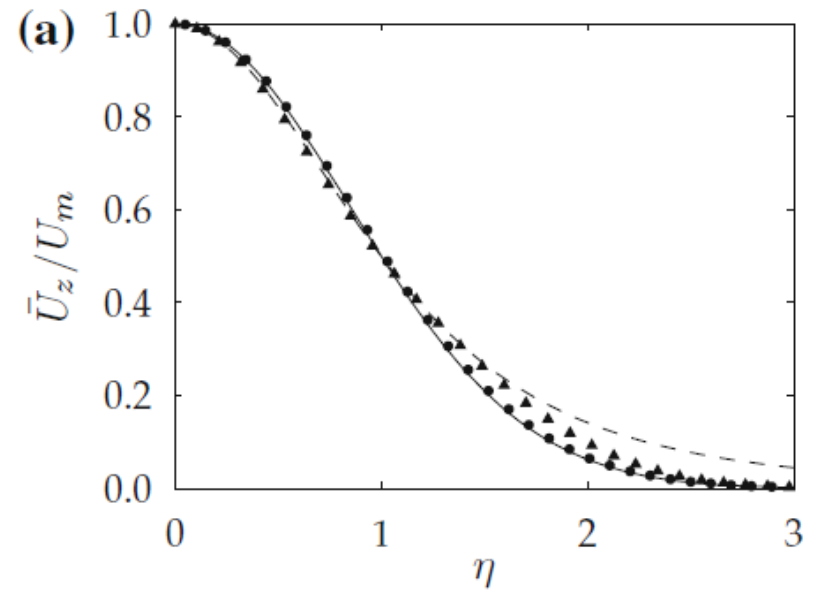
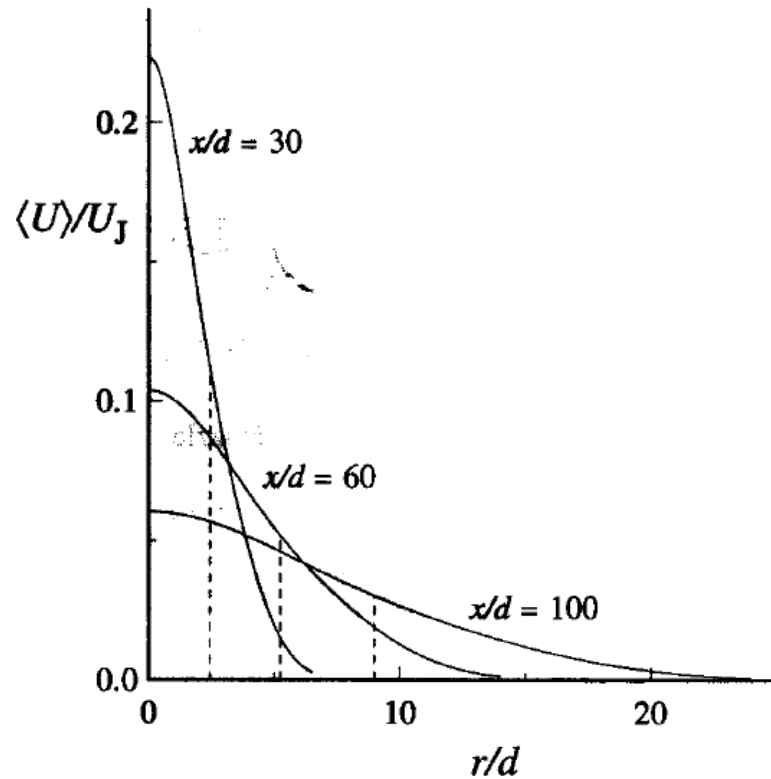
Turbine (RANS)



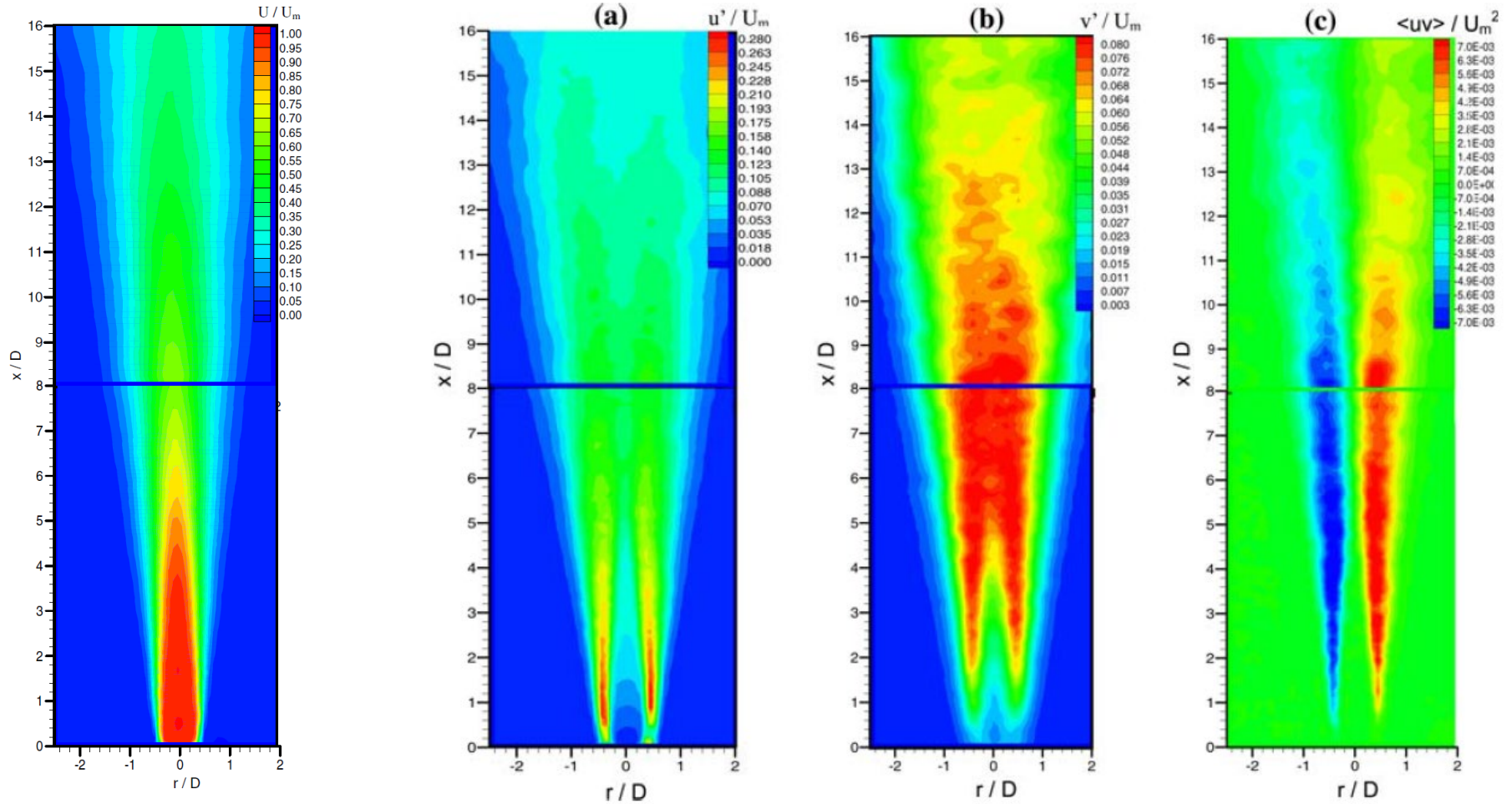
Jet turbulent : hypothèse de similitude



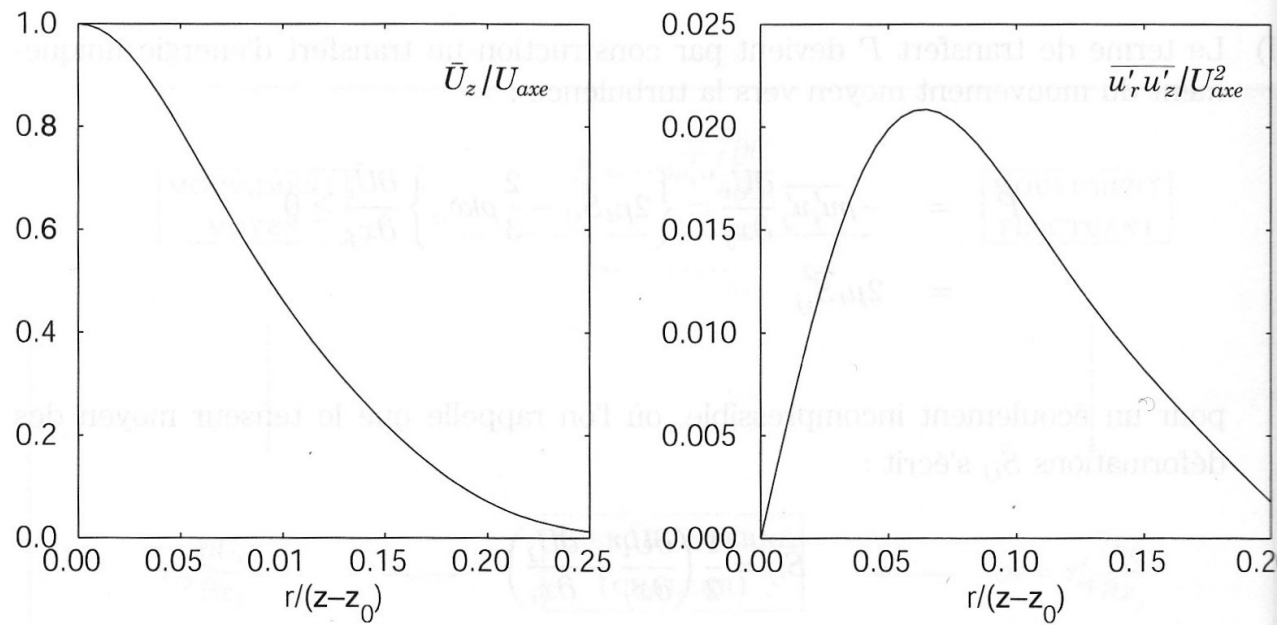
Jet turbulent : hypothèse de similitude



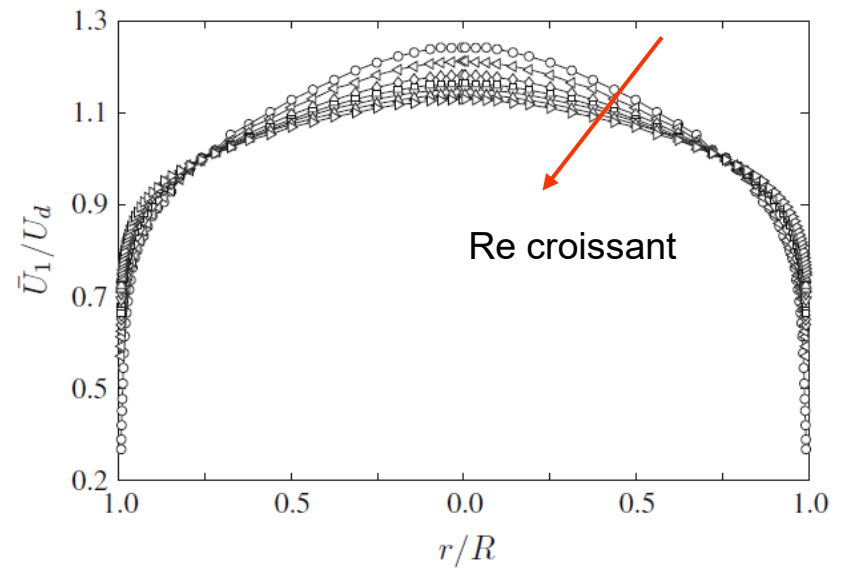
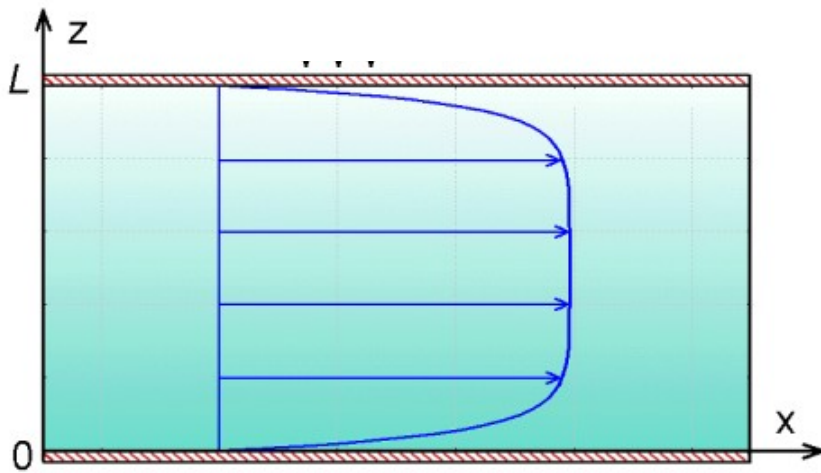
Jet turbulent : moyenne et contraintes de Reynolds



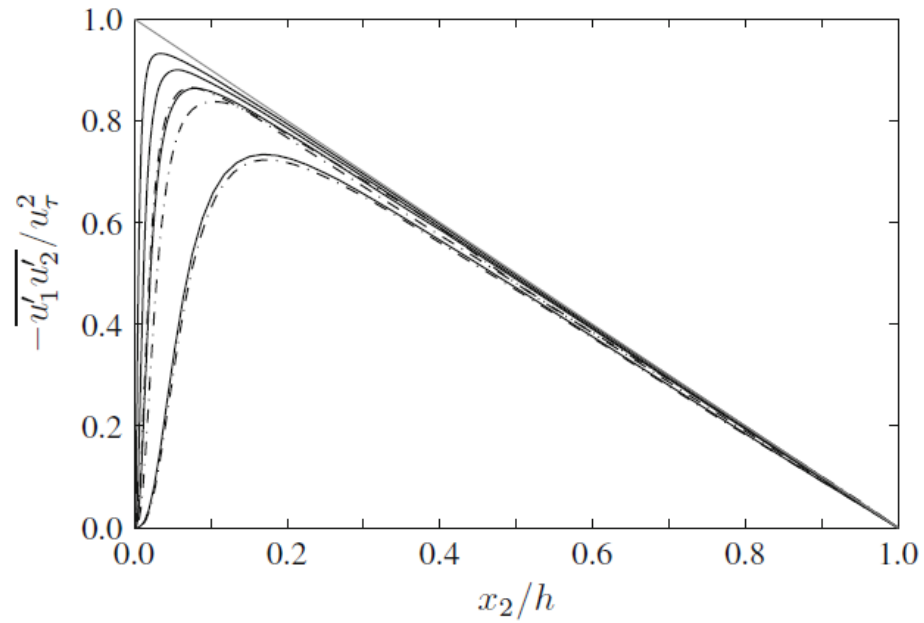
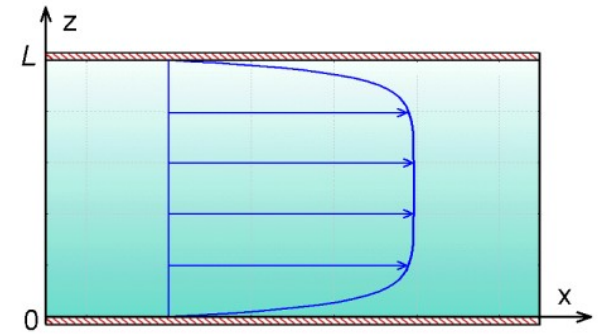
Jet turbulent : Tenseur de Reynolds



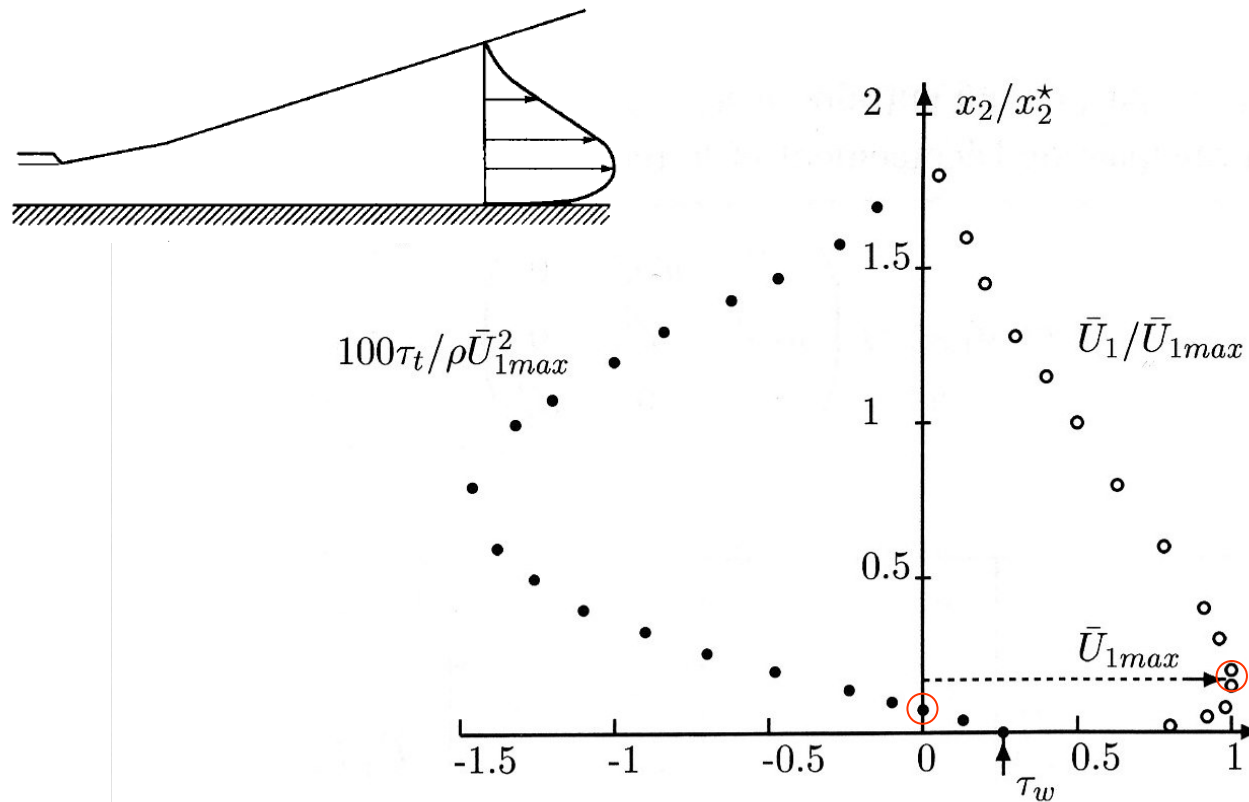
Ecoulement de canal



Canal plan : Contraintes de Reynolds

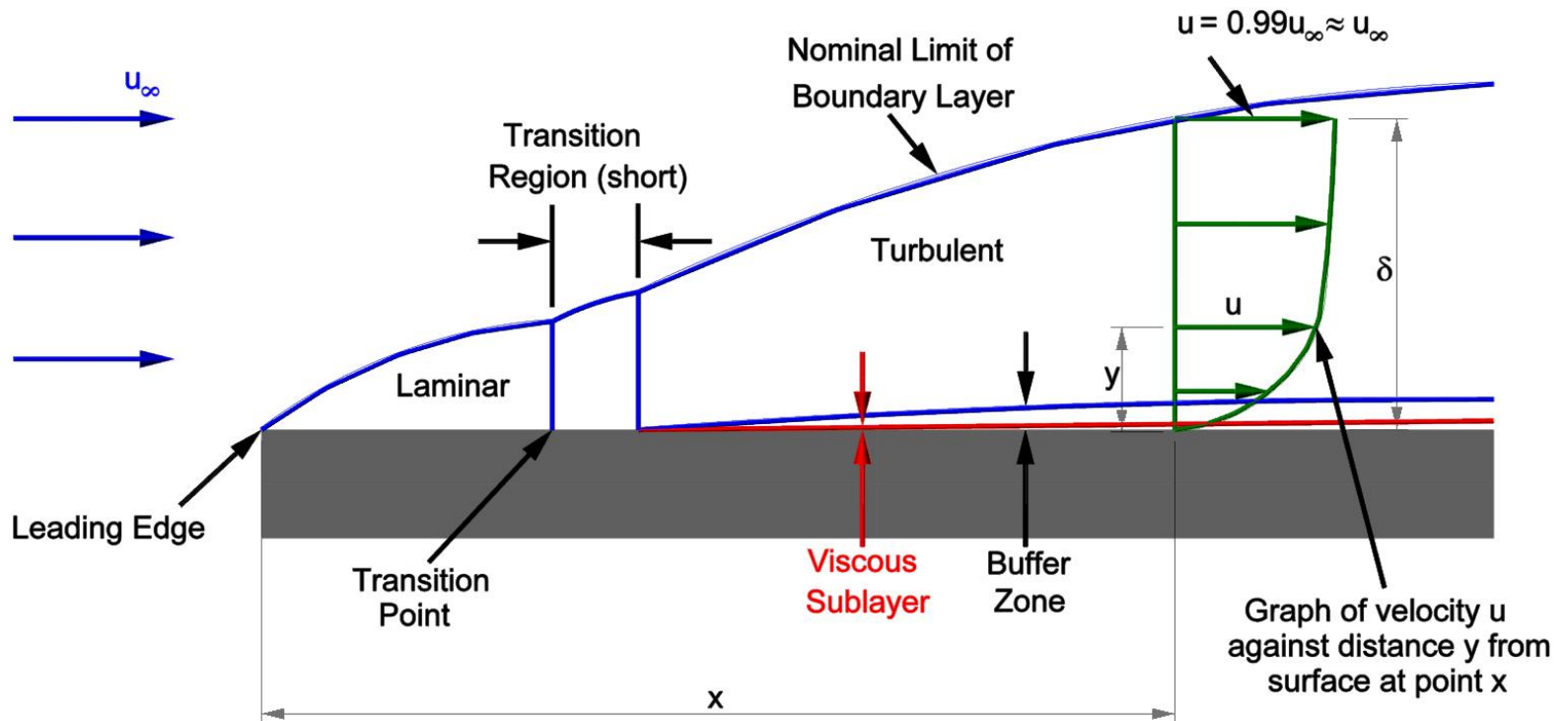
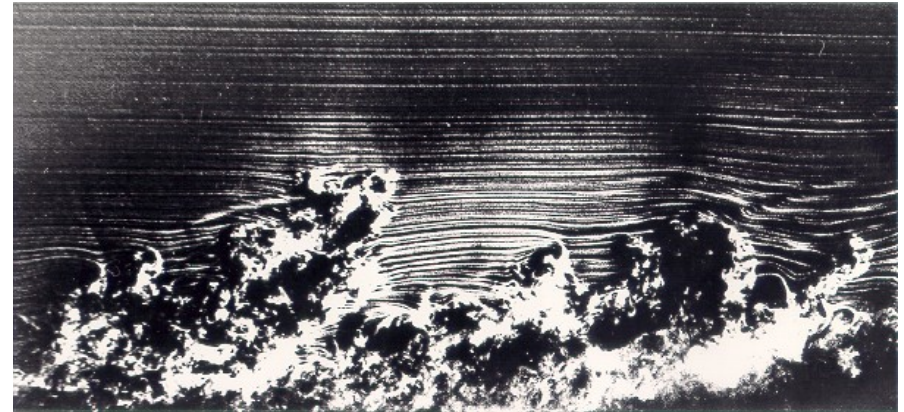


Contre-exemple : le jet pariétal invalidité modèles de viscosité turbulente (mixing length, $k-\epsilon$)



Jet plan pariétal, $Re \simeq 2 \times 10^4$: profils de la vitesse moyenne $\bar{U}_1$ et de la contrainte totale de cisaillement $\tau_t = \mu d\bar{U}_1/dx_2 - \rho\overline{u'_1 u'_2}$ en fonction de la distance à la paroi, x_2/x_2^* , où x_2^* est la distance à laquelle $U_1 = 0.5 \times U_{1max}$; τ_w est la tension à la paroi. En s'écartant de la paroi, on trouve d'abord le point où $\tau_t = 0$, puis celui où $d\bar{U}_1/dx_2 = 0$. D'après MATHIEU J.^{481, 485}.

Couche limite turbulente



Couche limite turbulente

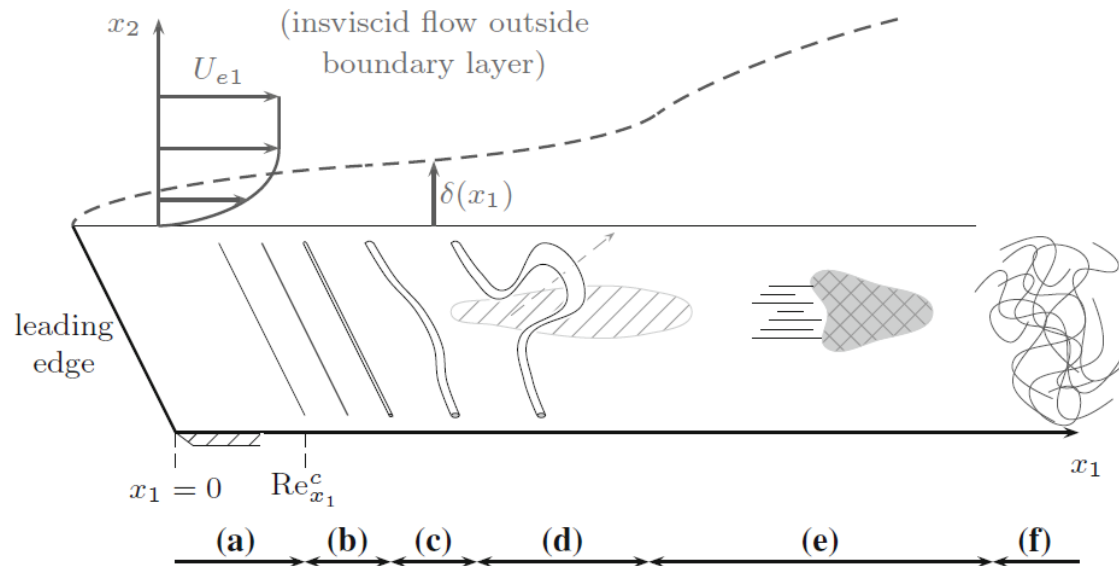


Fig. 1.6 Sketch of the transition of a laminar boundary layer developing over a flat plate with zero pressure gradient. (a) Stable laminar flow, (b) appearance of Tollmien-Schlichting instability waves, (c) nonlinear effects and tridimensional evolution, (d) large scale structures such as hairpin vortices, (e) growth of bursts and (f) fully turbulent boundary layer for $Re_{x_1} \geq 3 \times 10^6$

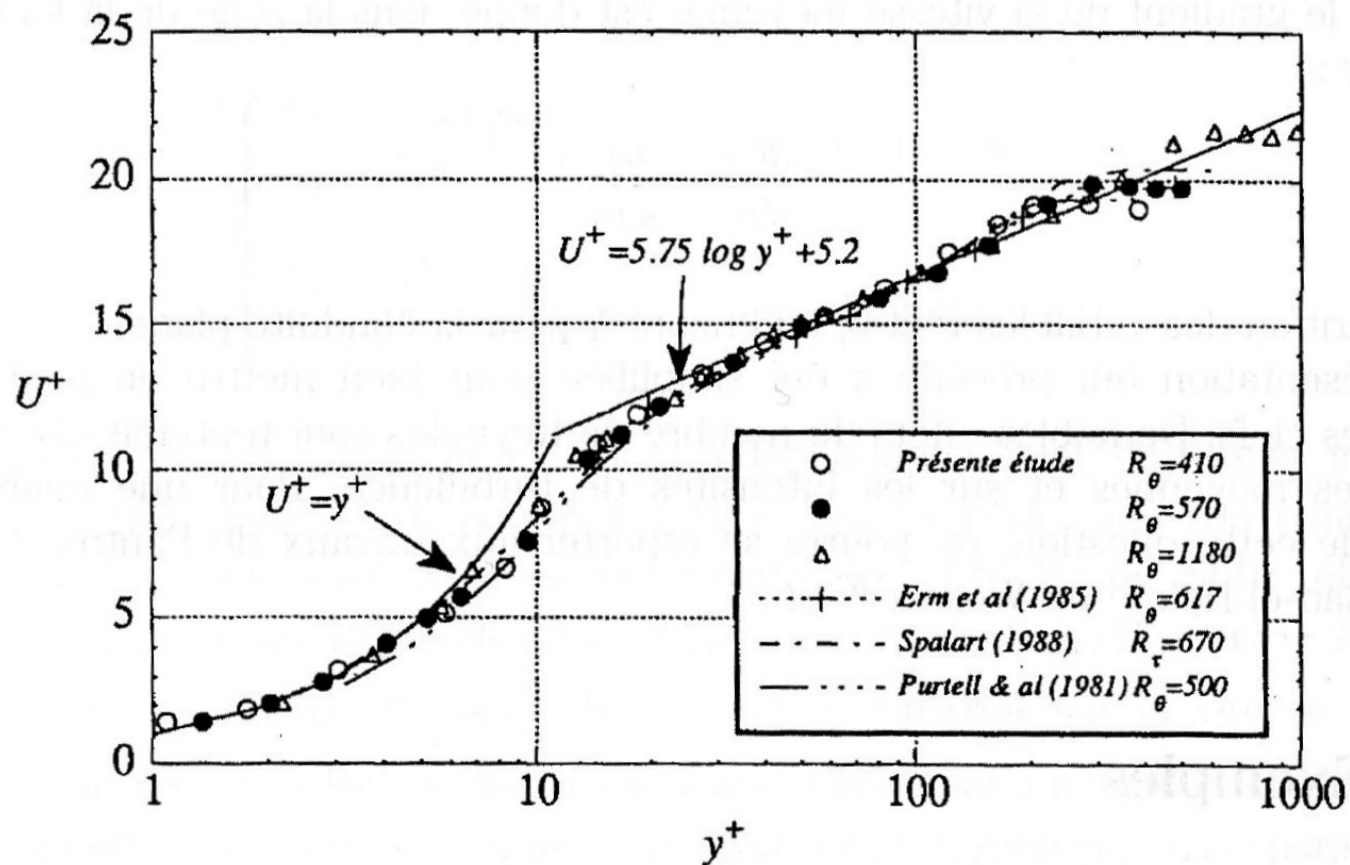
Turbulent boundary layer (experiment)



Turbulent boundary layer (DNS)

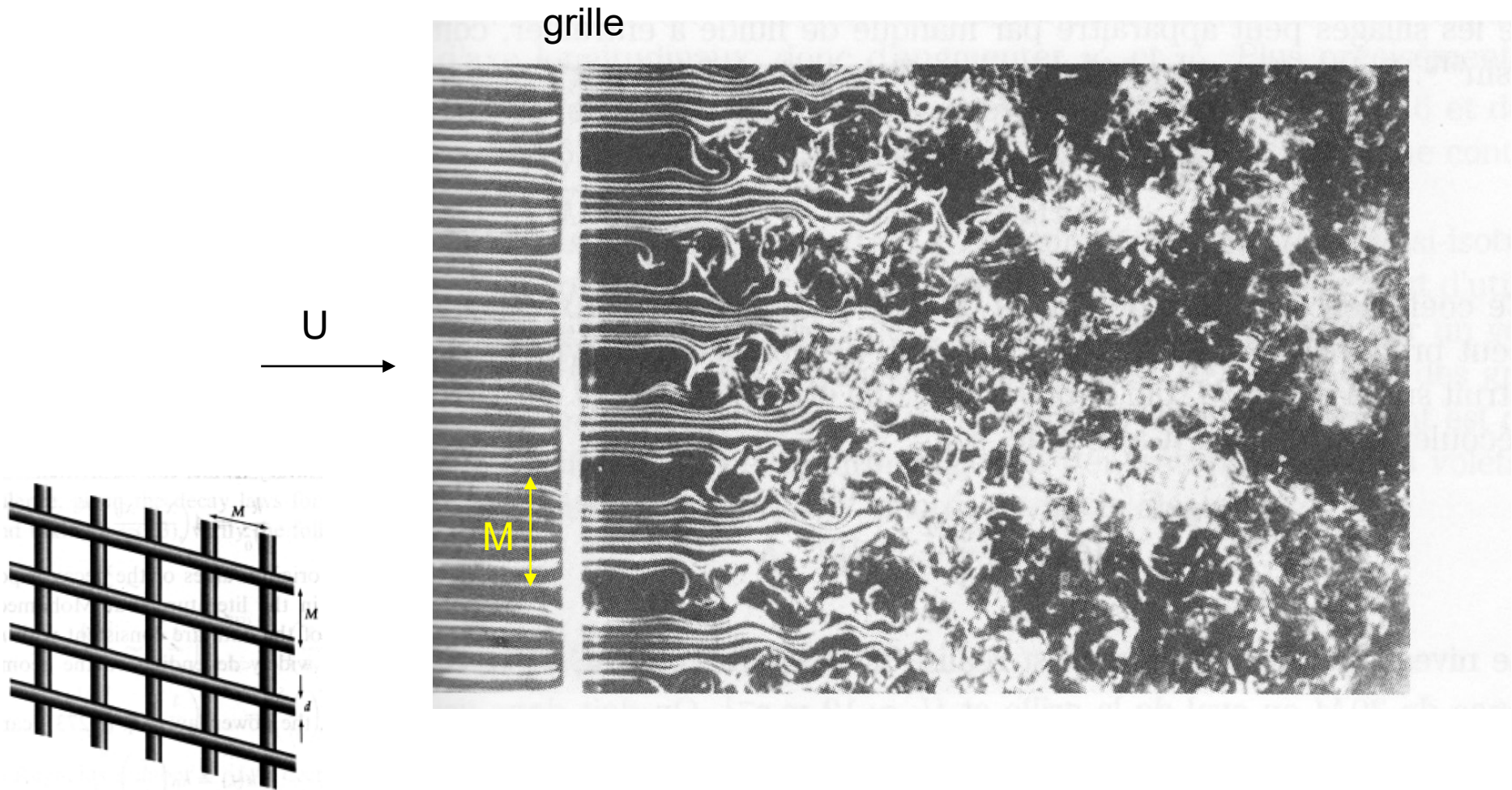


Couche limite turbulente



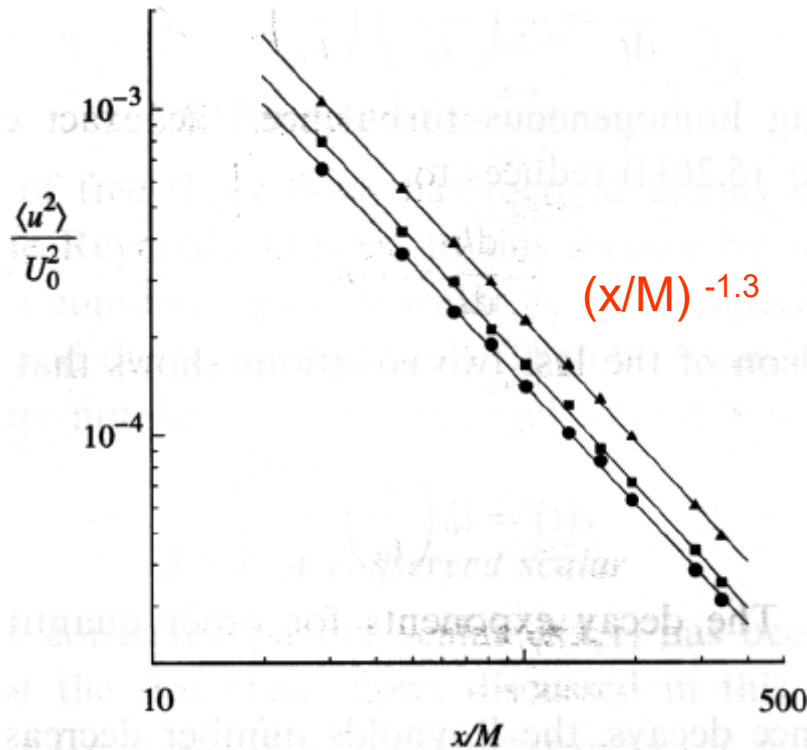
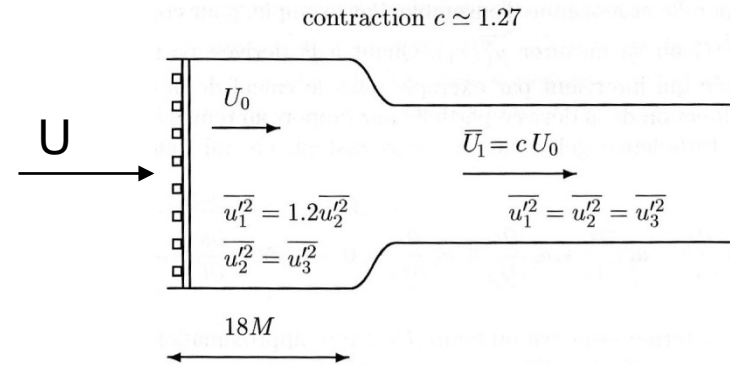
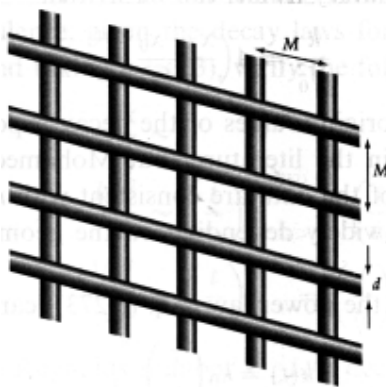
(Laadhari, d'après Bailly & Comte-Bellot)

Turbulence de grille (TD3)



D'après Corke & Nagib (Van Dyke)

Turbulence de grille



(Comte-Bellot & Corrsin, 1966)

Jet turbulent : Bilan d'énergie cinétique turbulente k

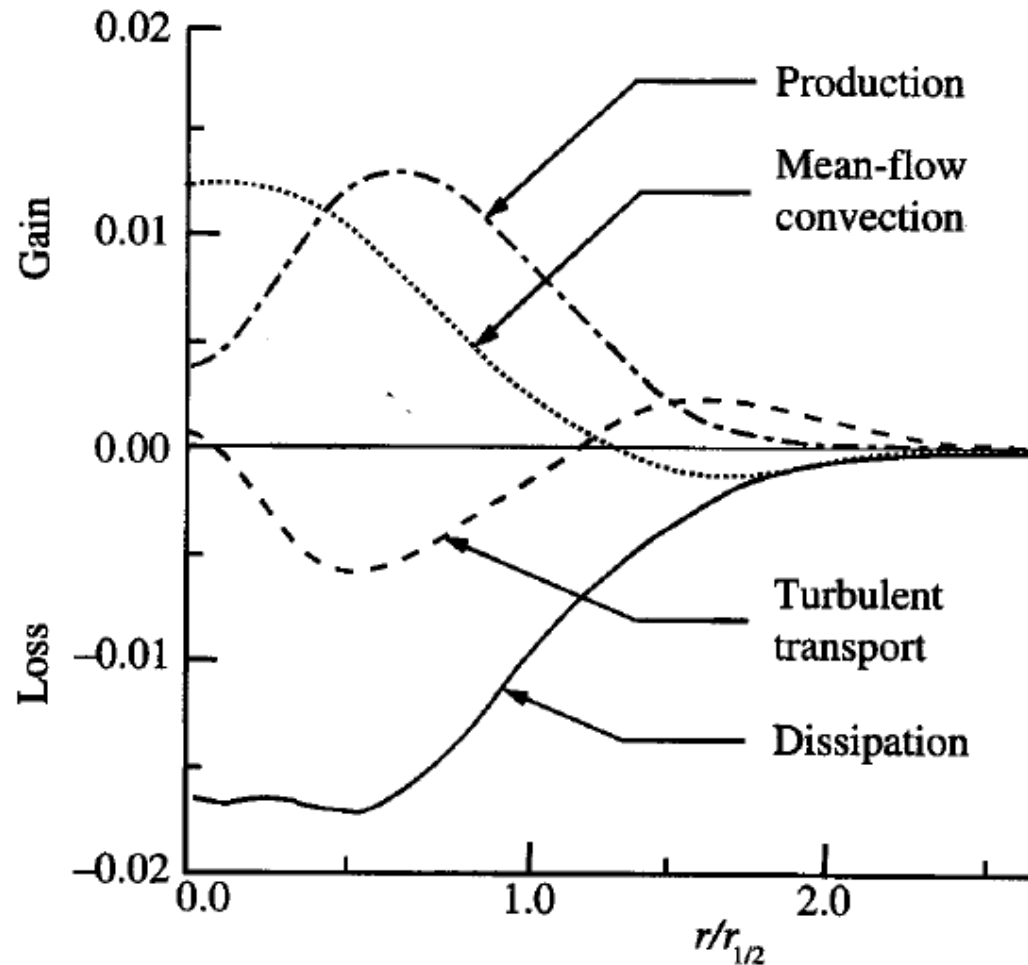


Fig. 5.16. The turbulent-kinetic-energy budget in the self-similar round jet. Quantities are normalized by U_0 and $r_{1/2}$. (From Panchapakesan and Lumley (1993a).)

Codes CFD (computational fluid dynamics)

Codes industriels courants :

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CD-adapco (Star-CD)

FlowScience (Flow3D)



Viscous Model

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- ☐ Inviscid
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- ☒ Standard Wall Functions
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- ☐ User-Defined Wall Functions

Options

- ☐ Viscous Heating

Model Constants

Cmu: 0.09

C1-Epsilon: 1.44

C2-Epsilon: 1.92

TKE Prandtl Number: 1

User-Defined Functions

Turbulent Viscosity: none

Prandtl Numbers

TKE Prandtl Number: none

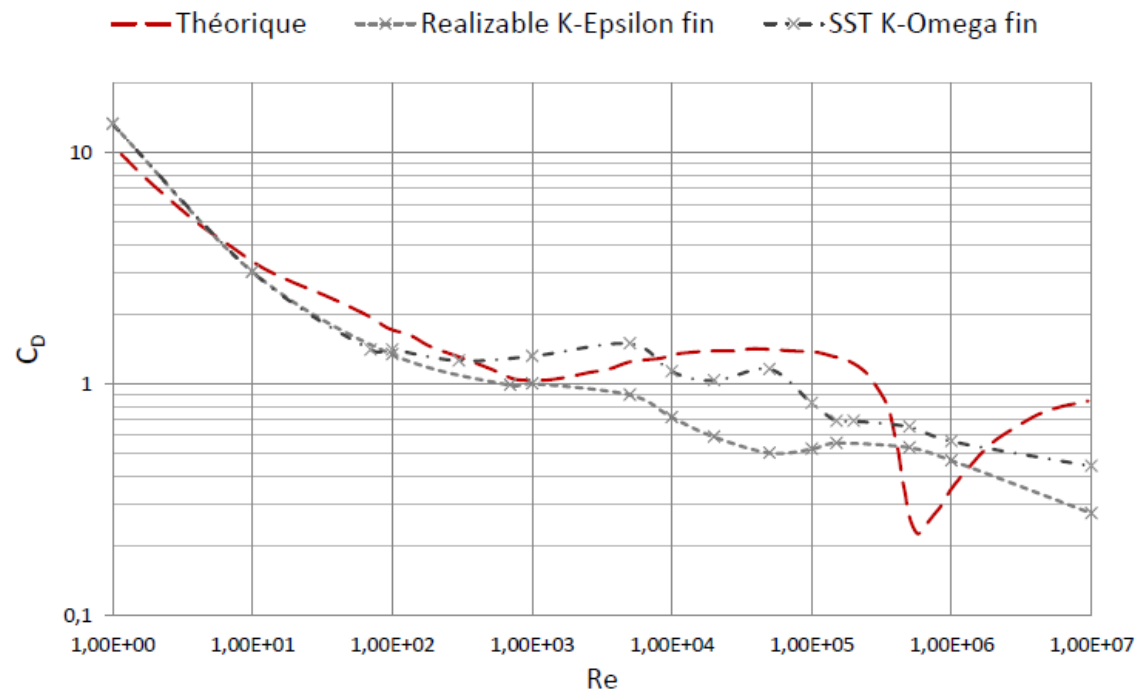
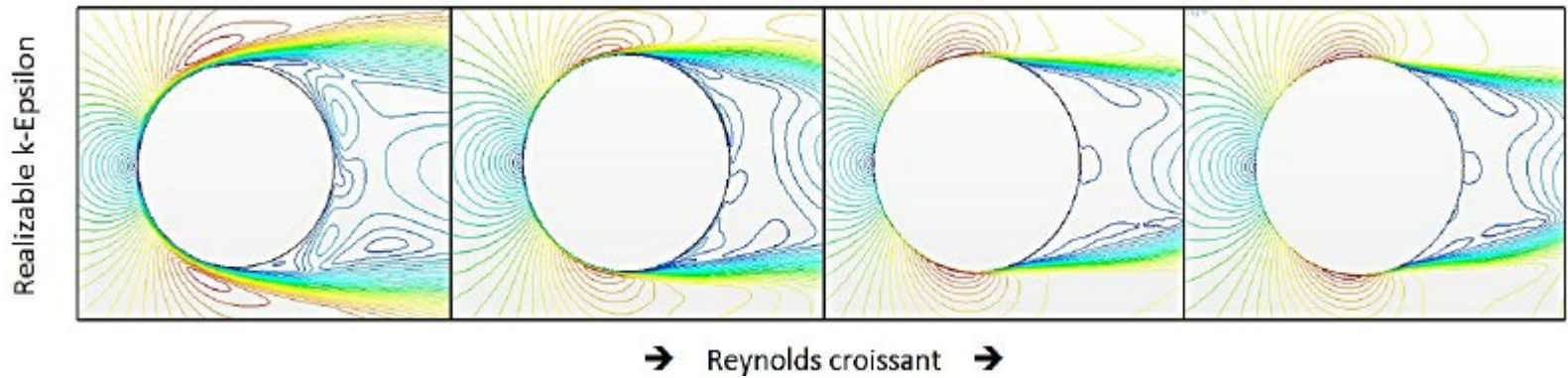
TDR Prandtl Number: none

Energy Prandtl Number: none

OK Cancel Help

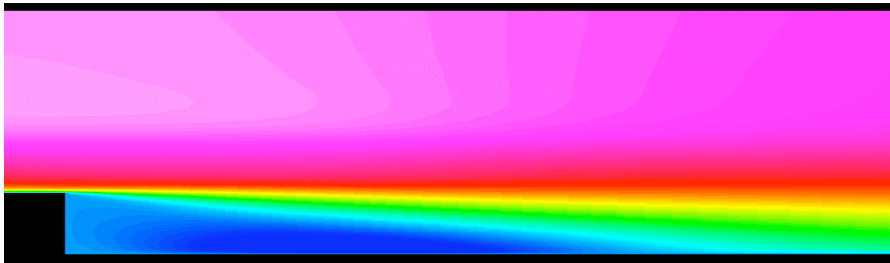
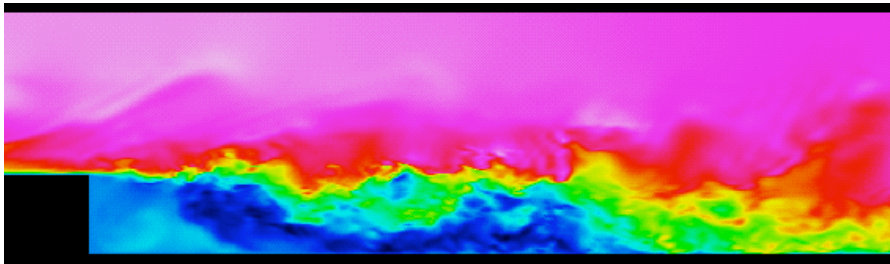
(Fluent)

Sillage d'un cylindre : modèle k-epsilon



Données
D. Etien /
Alstom (2012)

Ex: Marche descendante



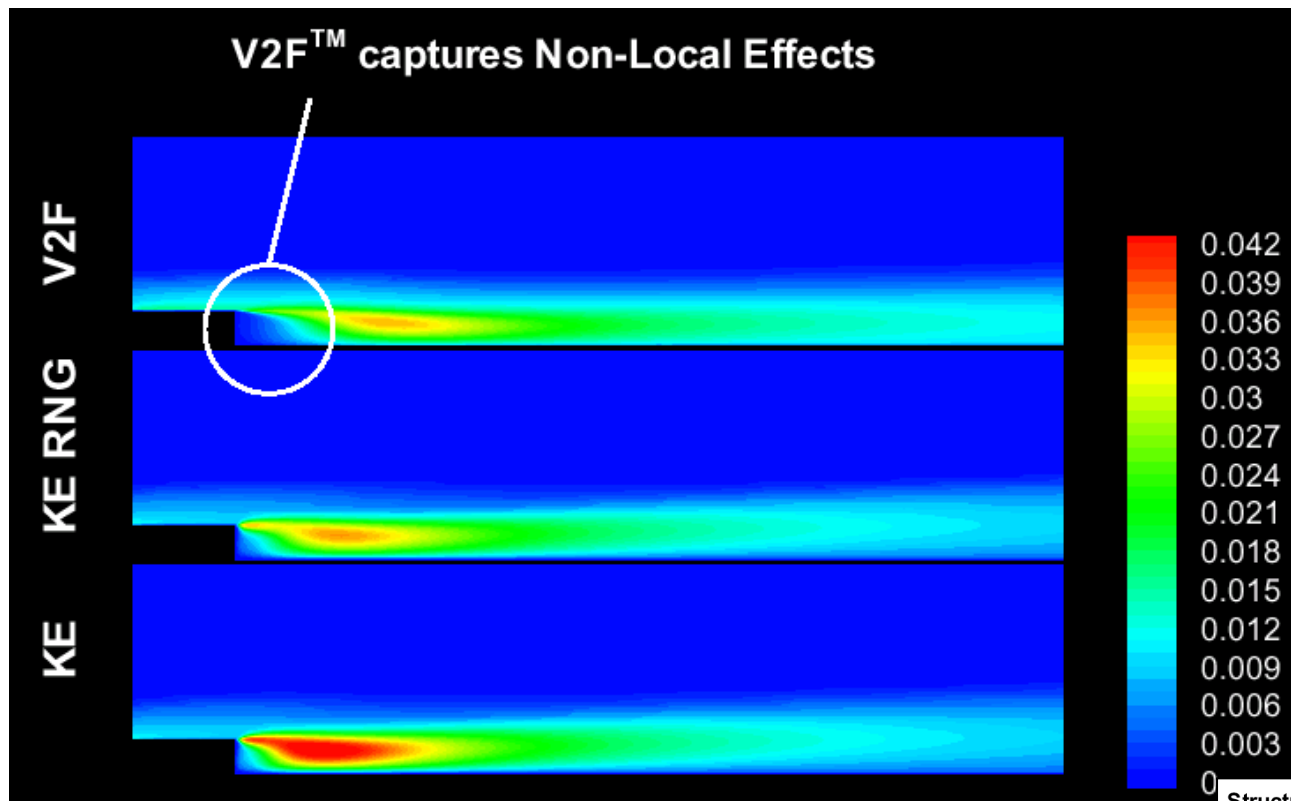
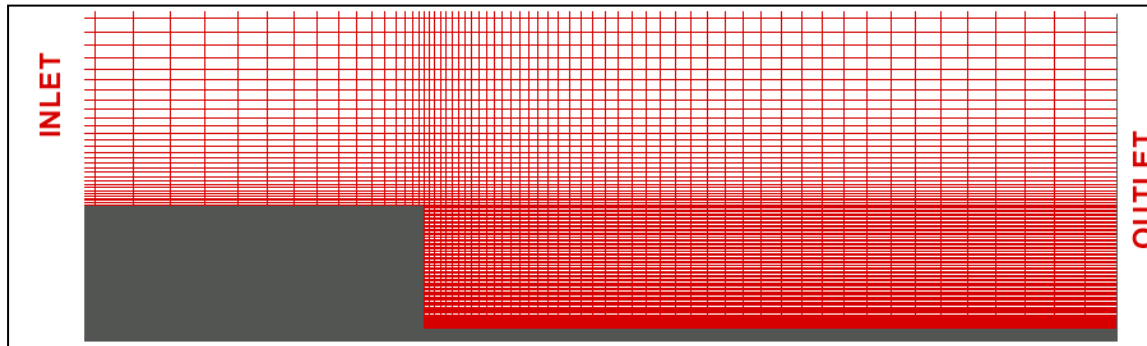
Reynolds number: 5100

Expansion ratio: 1.2

Grid: 768 x 192 x 64

***Normalization based on
step height H and
free-stream velocity at inlet***

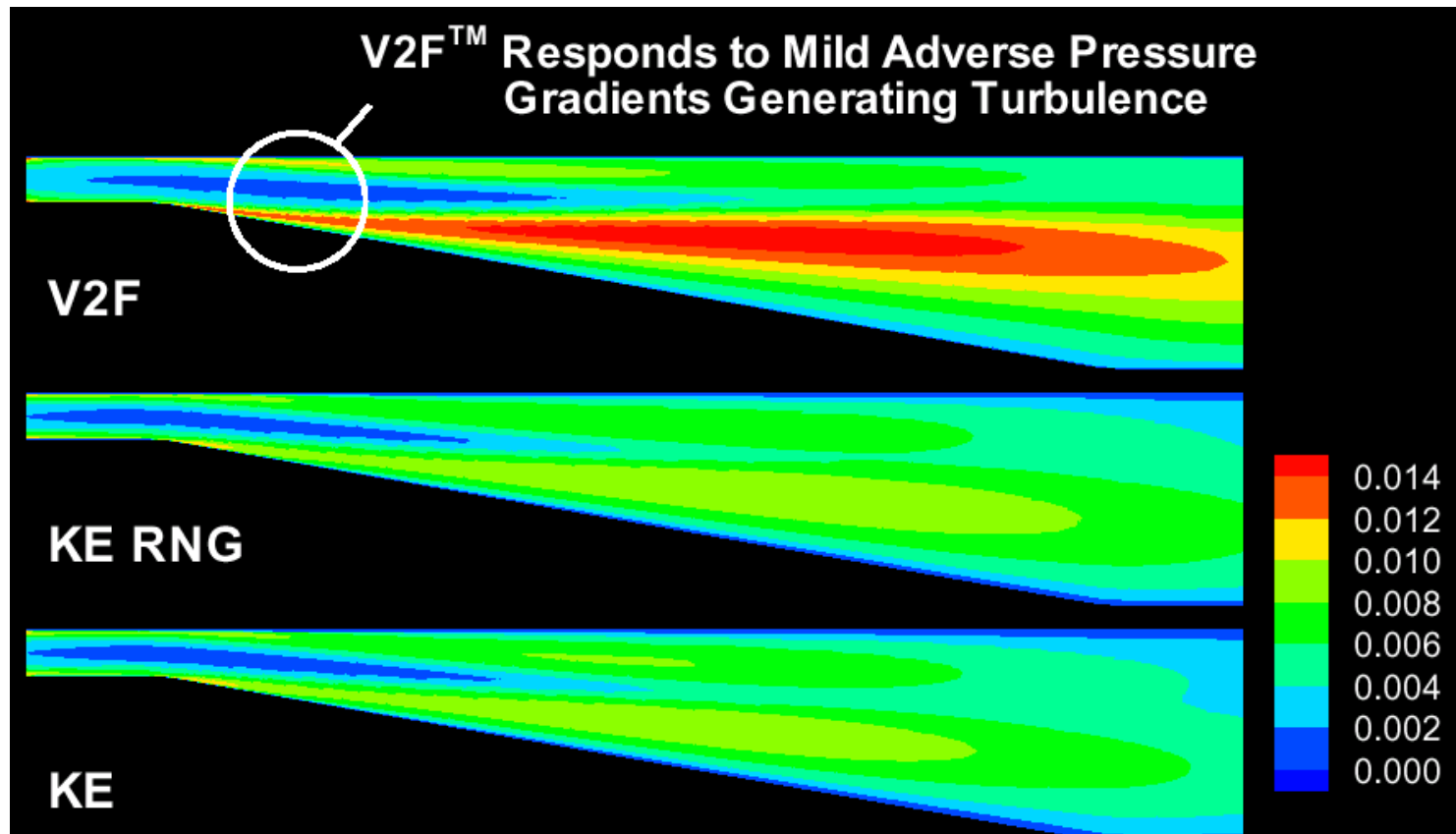
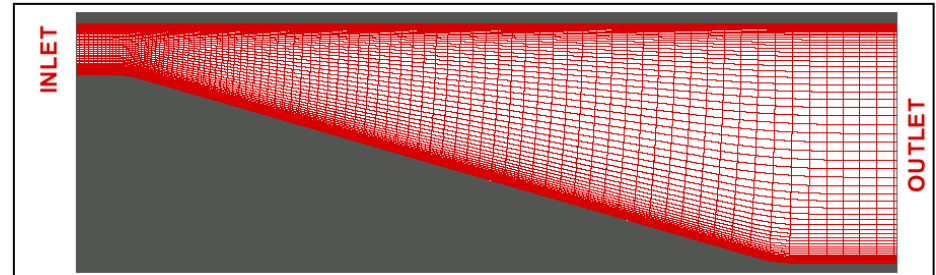
Ex: Marche descendante



Structured Grid (10000 cells)
Reynolds Number 5100
Fully Developed Turbulent Flow at the Inlet

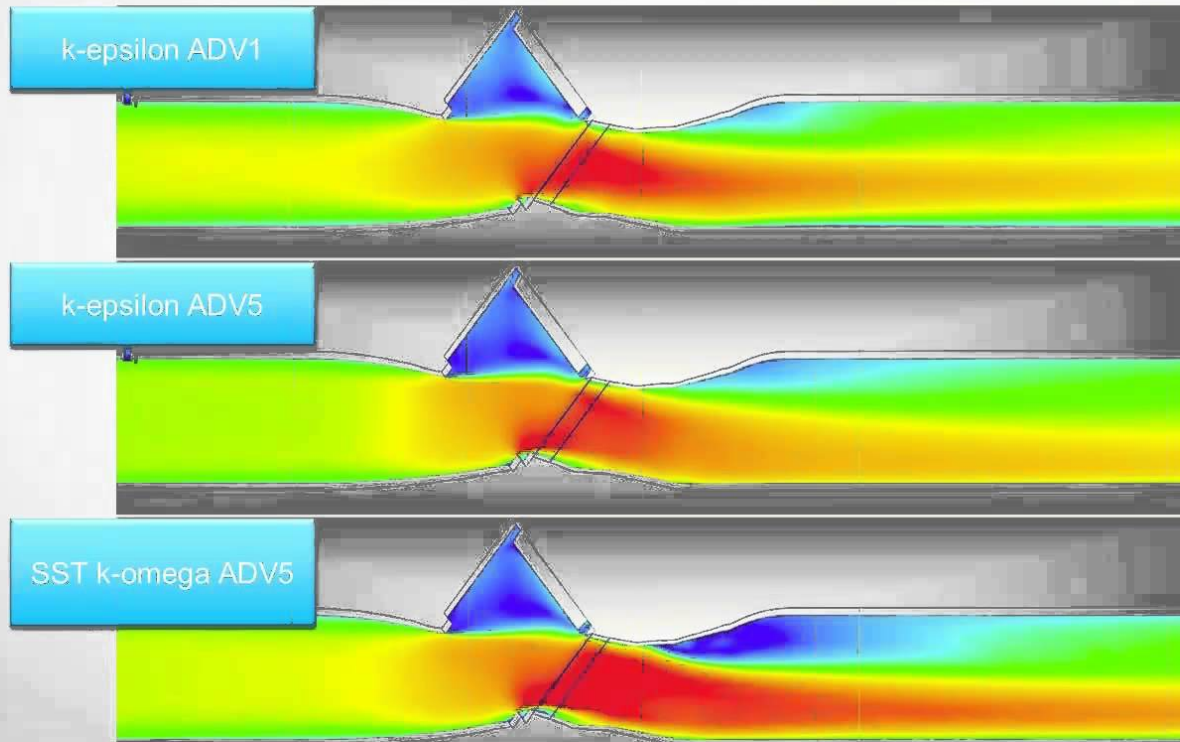
Modele V2F: Diffuseur asymmetrique

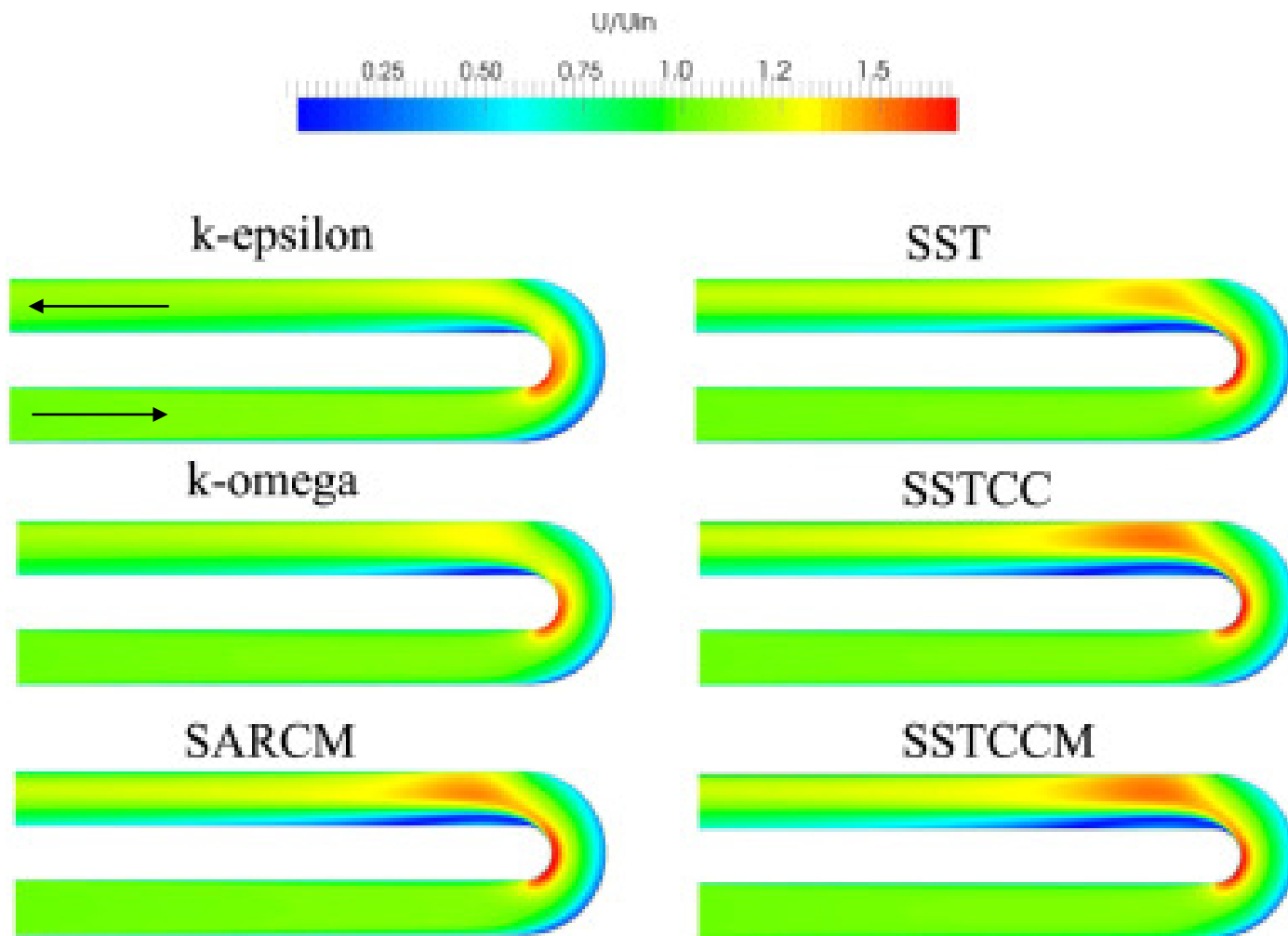
Structured Grid (10000 cells)
Reynolds Number 10000
Fully Developed Turbulent Flow at the Inlet



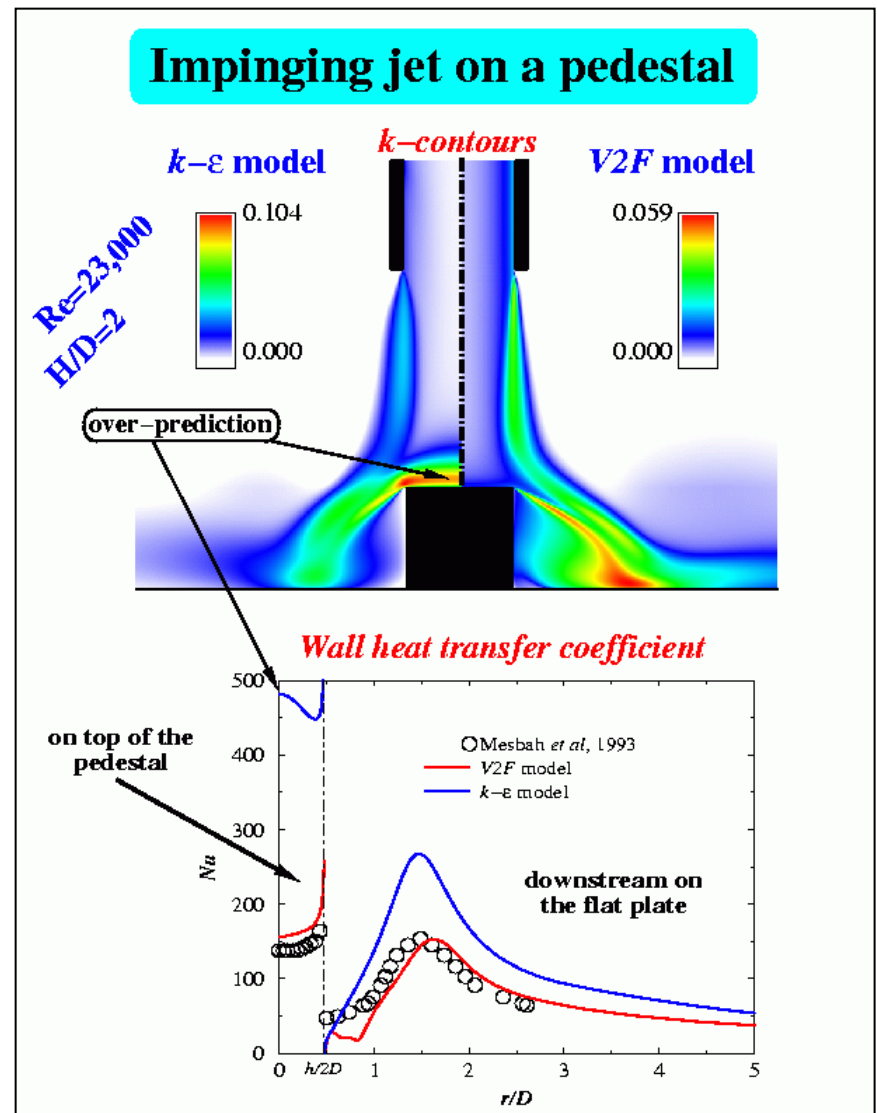
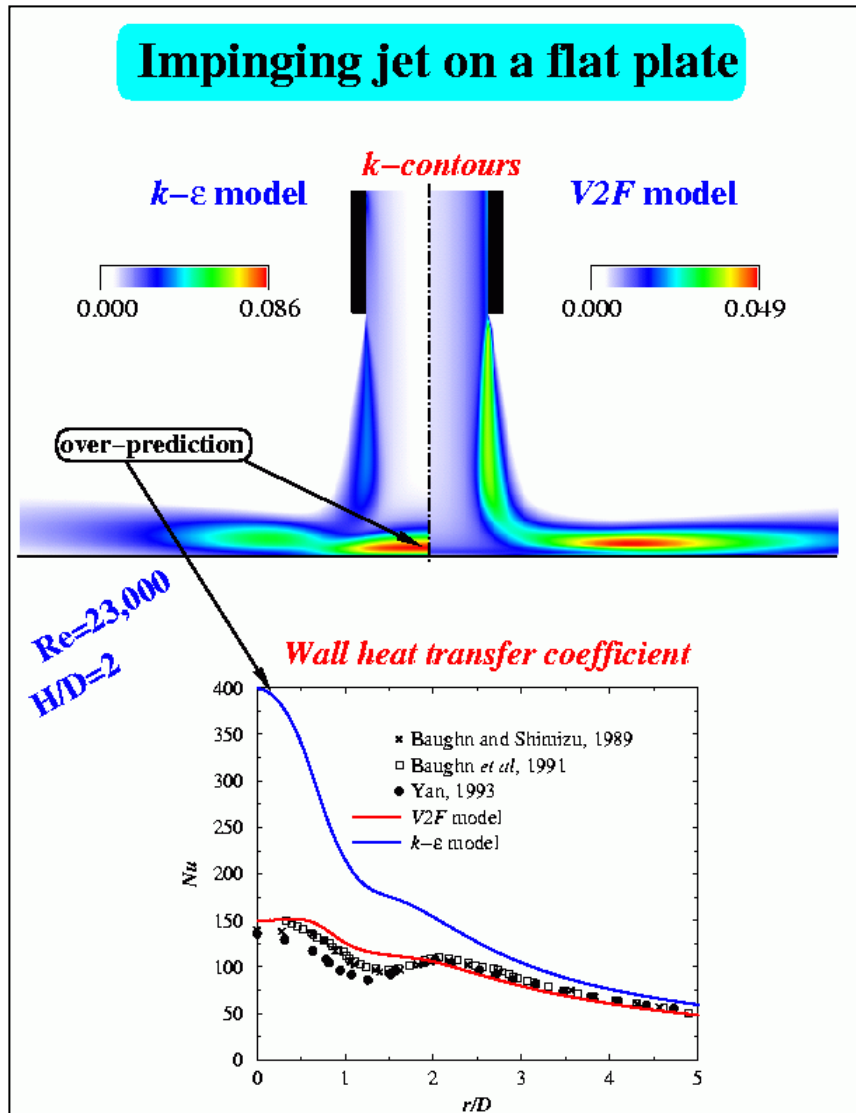
K-omega SST

Check Valve Example





Jet impactant (impinging jet) - modèle V2F



Modèle k - ε "bas-Reynolds"

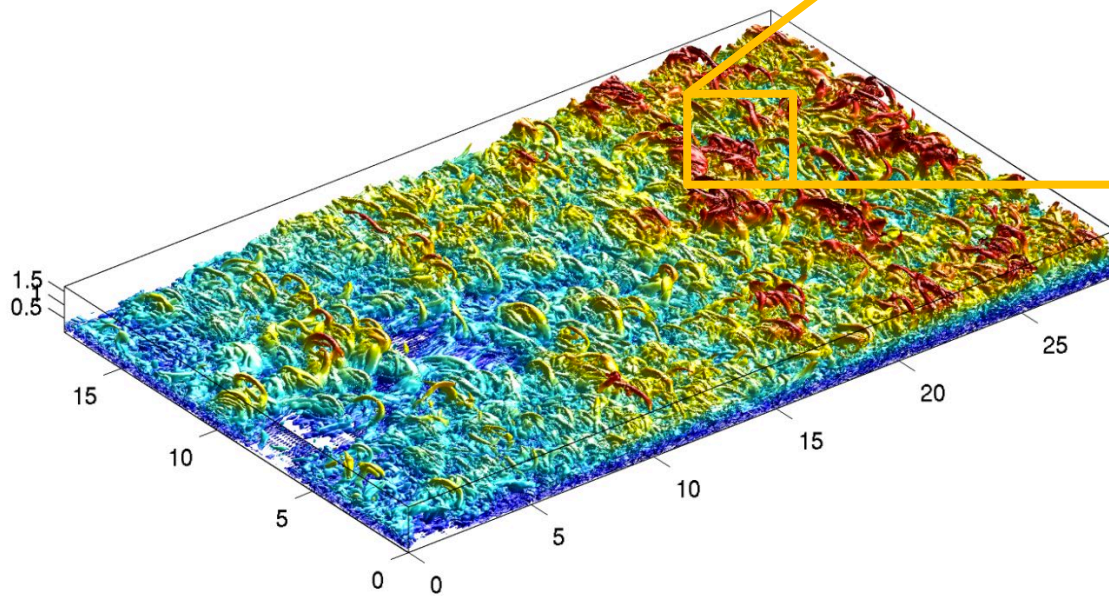
$$\nu_t = c_\mu f_\mu \frac{k^2}{\varepsilon}, \quad P = -\overline{u_i' u_j'} \frac{\partial \bar{u}_i}{\partial x_j}$$

$$\frac{D}{Dt} k = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_K} \right) \frac{\partial}{\partial x_j} k \right] + P - \varepsilon$$

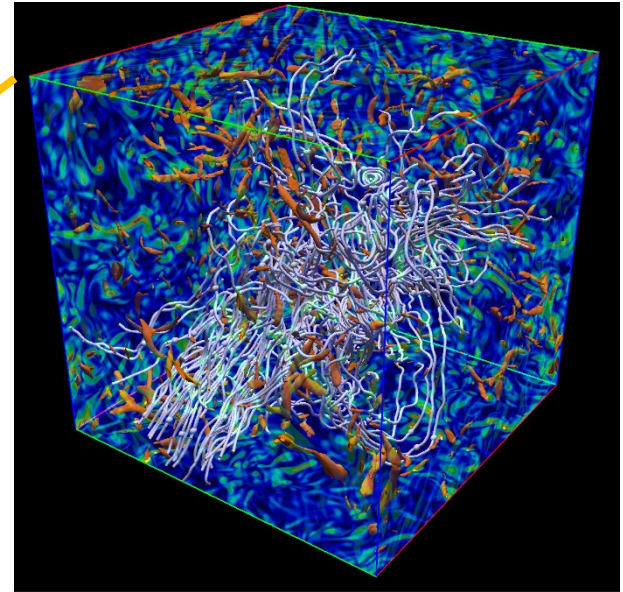
$$\frac{D}{Dt} \varepsilon = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial}{\partial x_j} \varepsilon \right] + \frac{\varepsilon}{k} c_{\varepsilon 1} f_1 P - c_{\varepsilon 2} f_2 \frac{\varepsilon^2}{k}$$

Exemple : Jones – Launder – Sharma :

$$f_1 = 1, f_2 = 1 - 0.3 \exp(-\tilde{\text{Re}}_t^2), f_\mu = \exp\left(\frac{-3.4}{(1 + \tilde{\text{Re}}_t/50)^2}\right), \tilde{\text{Re}}_t = \frac{k^2}{\nu \varepsilon}$$



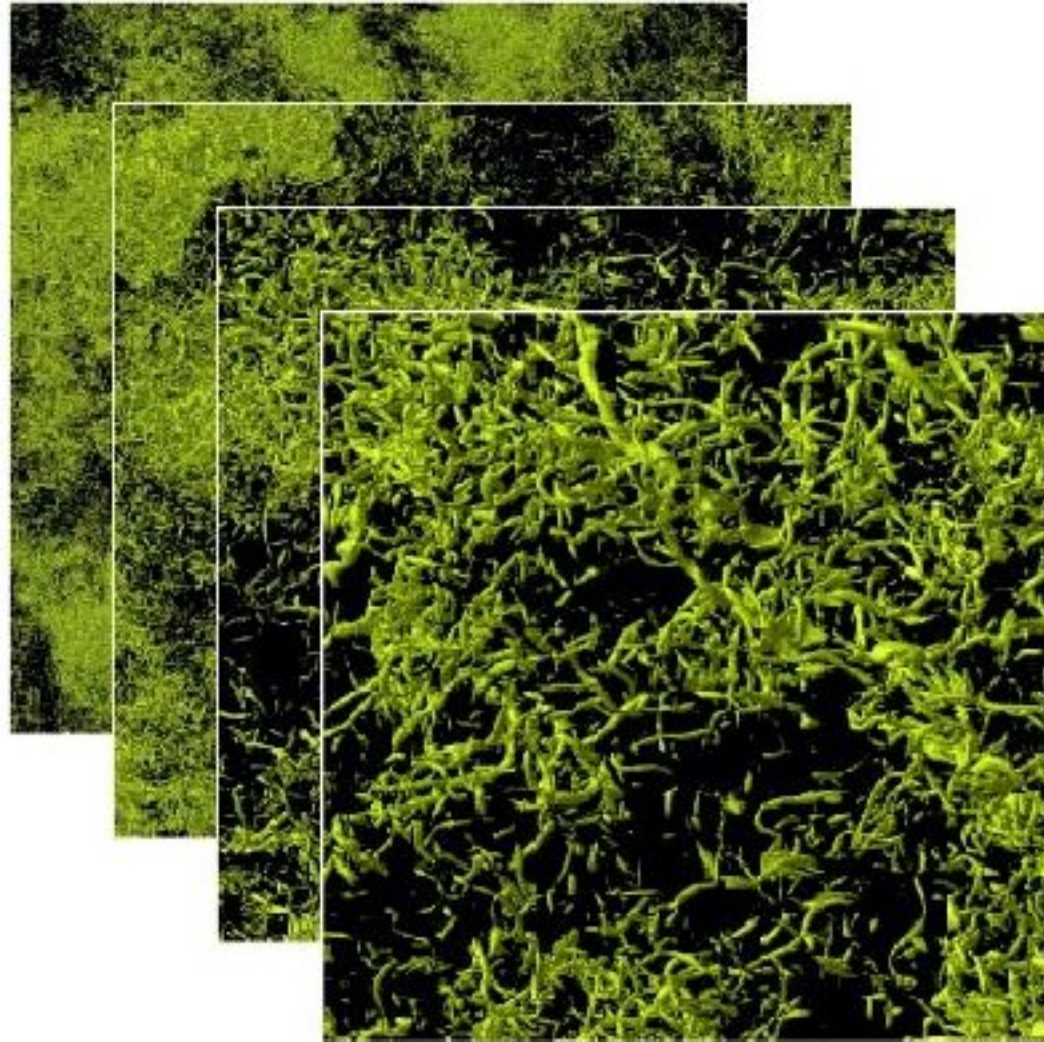
?



Turbulence isotrope
(V. Calo, Univ. Texas)

Couche limite turbulente
(DNS Sillero & Jimenez, 2011)

Earth Simulator : DNS 4096^3 , $R_\lambda = 1130$



Kaneda, Univ Nagoya, Japan



Entry #: 84174

Vortices within vortices: hierarchical nature of vortex tubes in turbulence

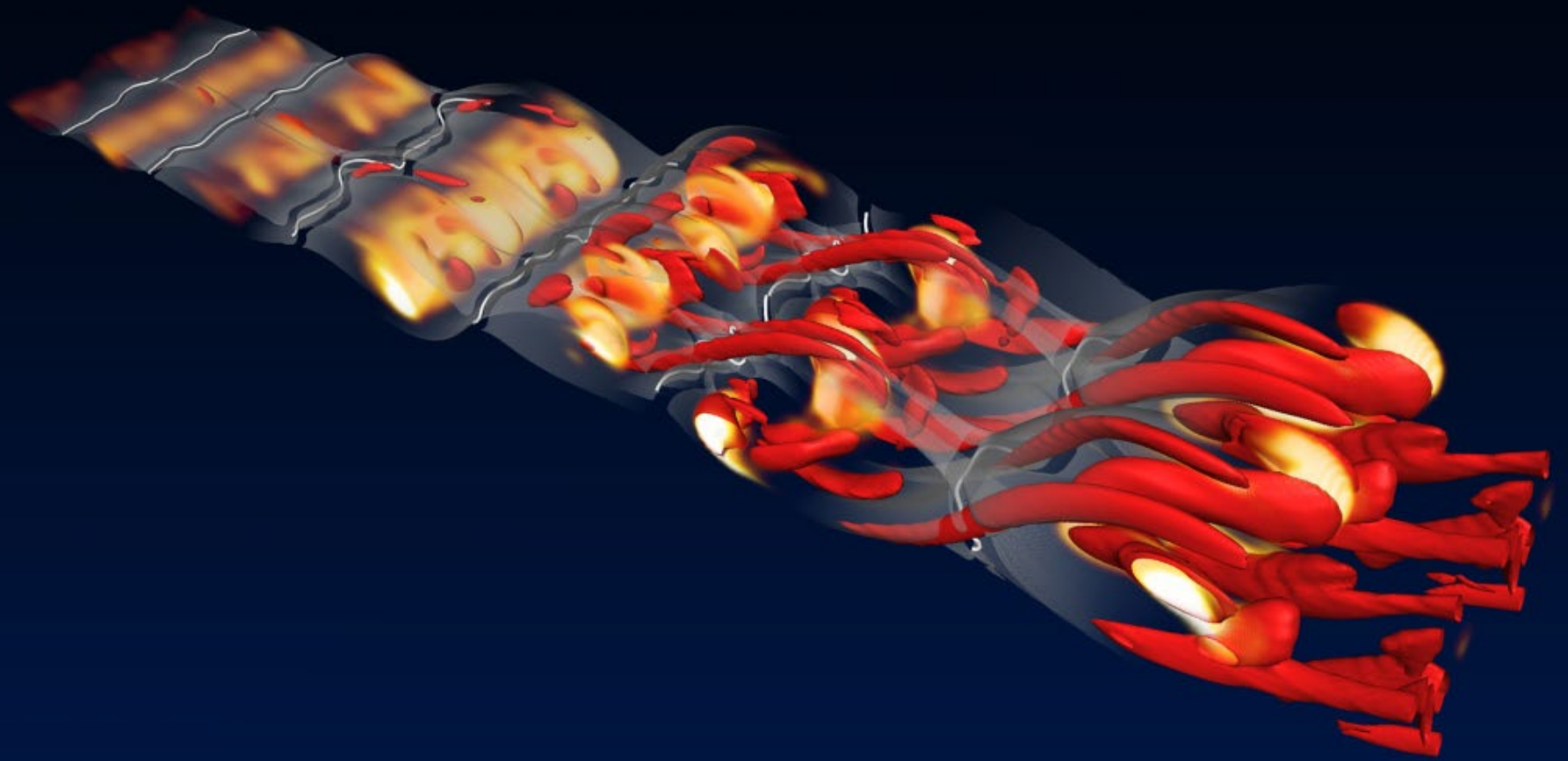
Kai Bürger¹, Marc Treib¹, Rüdiger Westermann¹,
Suzanne Werner², Cristian C Lalescu³,
Alexander Szalay², Charles Meneveau⁴, Gregory L Eyink^{2,3,4}

¹ Informatik 15 (Computer Graphik & Visualisierung), Technische Universität München

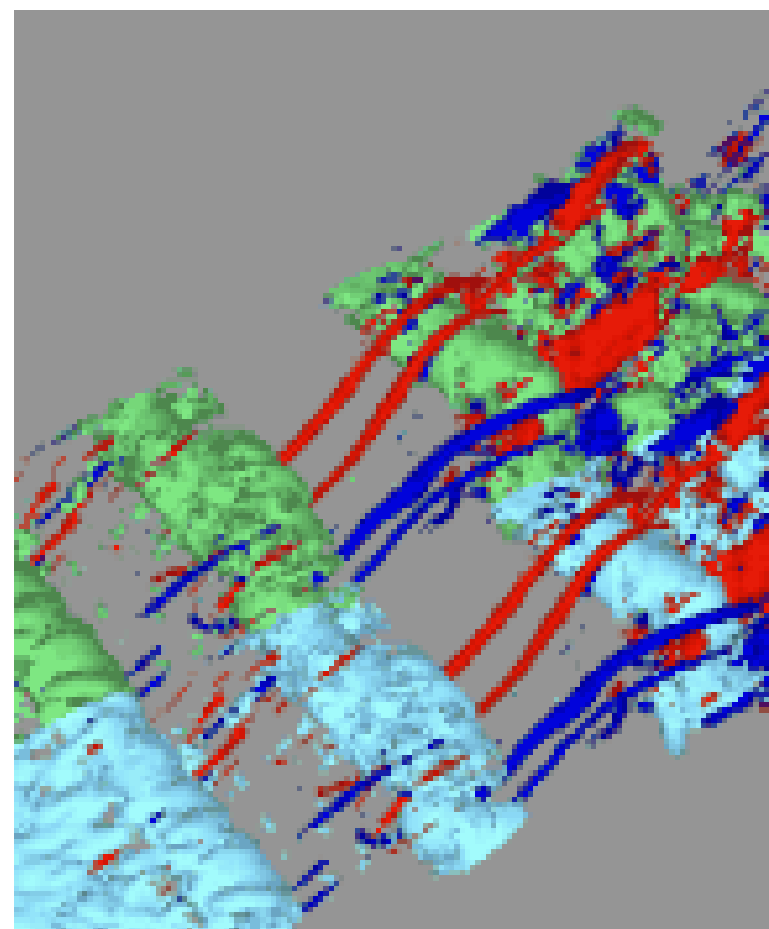
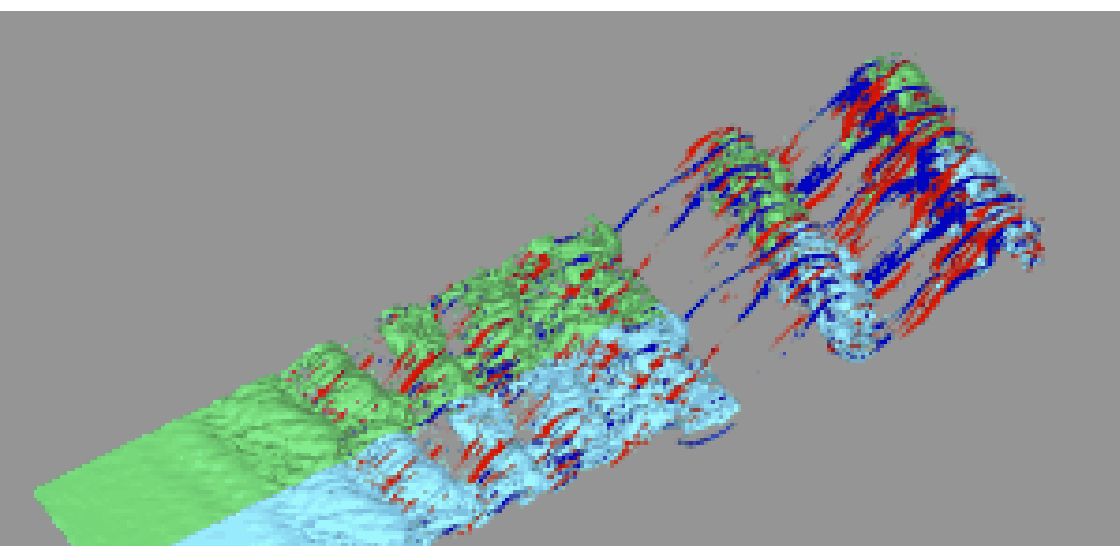
² Department of Physics & Astronomy, The Johns Hopkins University

³ Department of Applied Mathematics & Statistics, The Johns Hopkins University

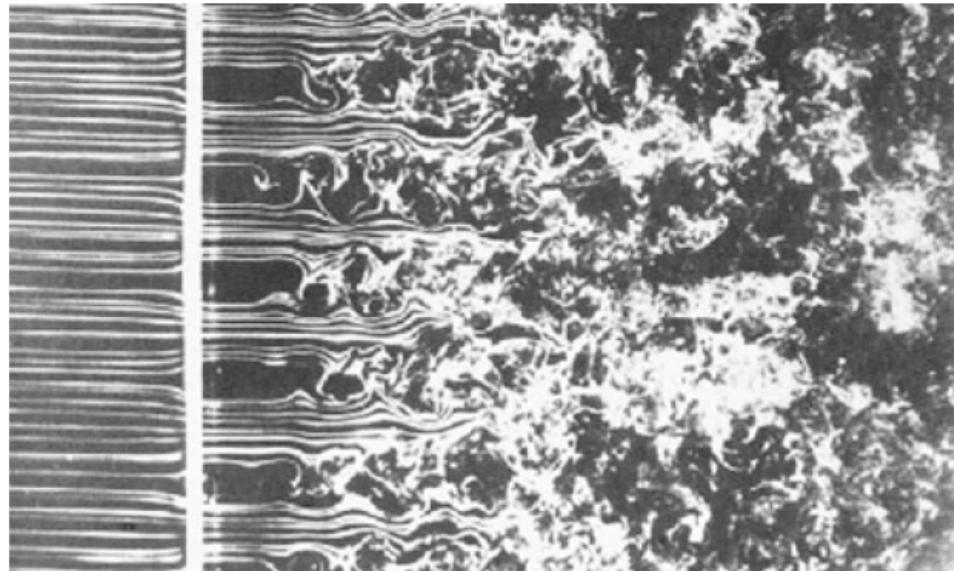
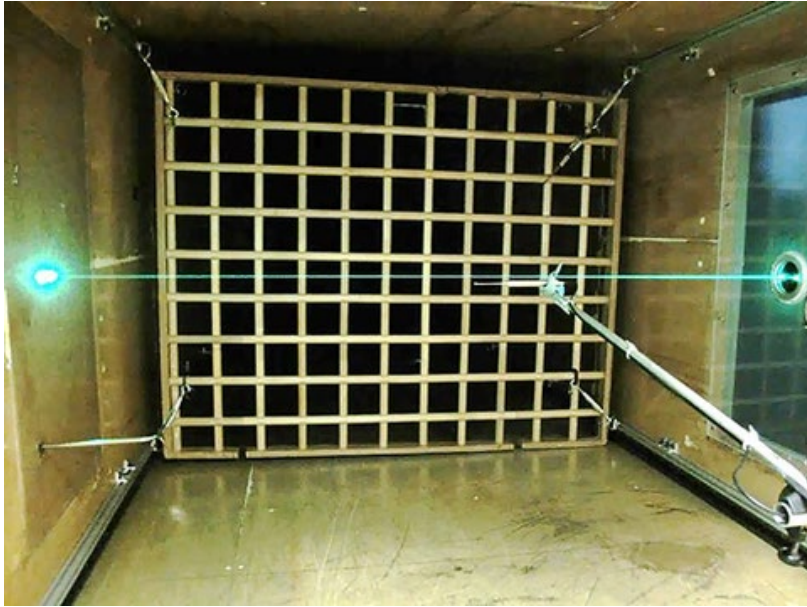
⁴ Department of Mechanical Engineering, The Johns Hopkins University



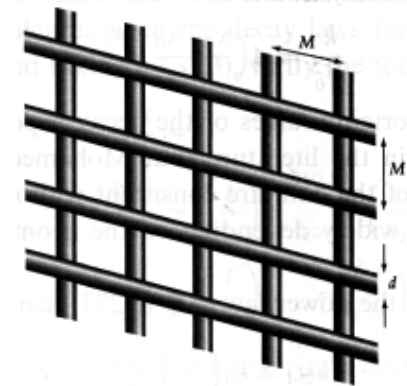
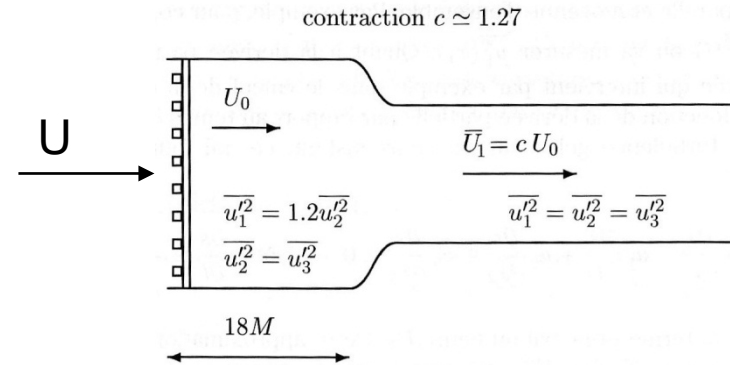
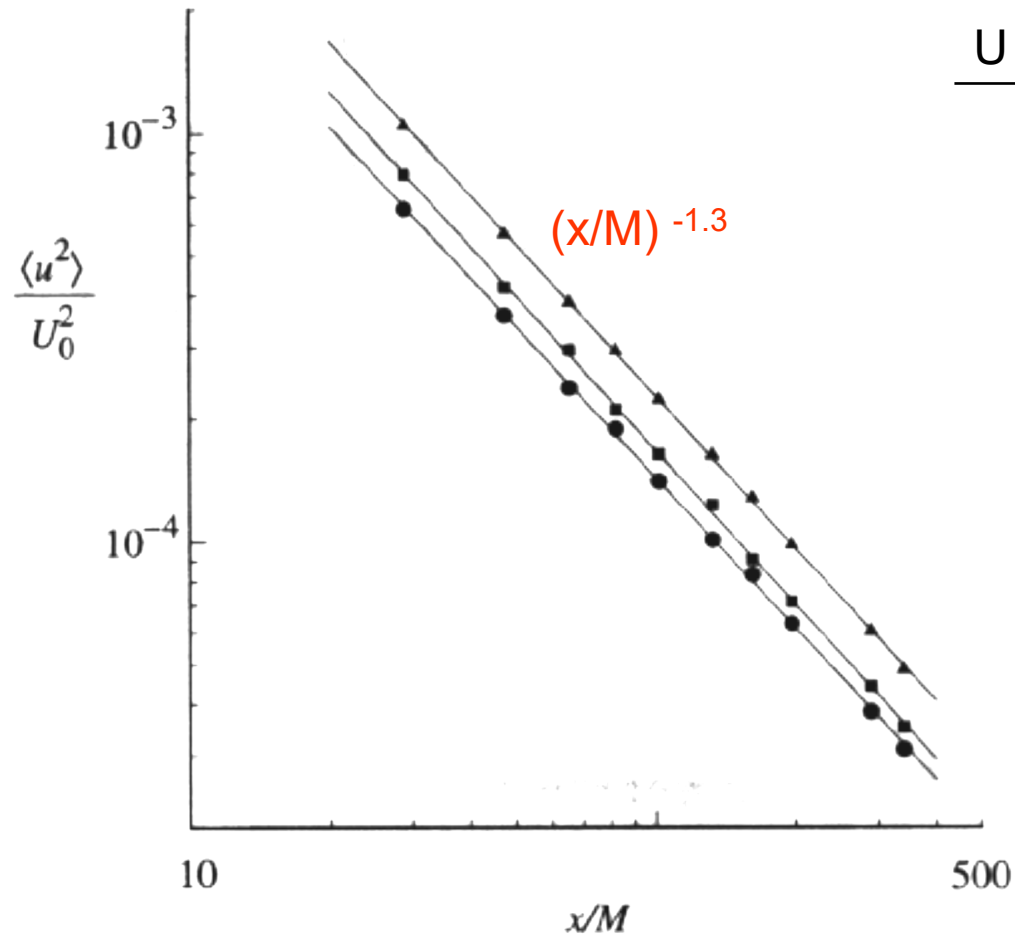
© 2004 Weinkauff (ZIB), Noack (TUB), Comte (ULP), Dillmann (DLR), Hege (ZIB)



Turbulence de grille



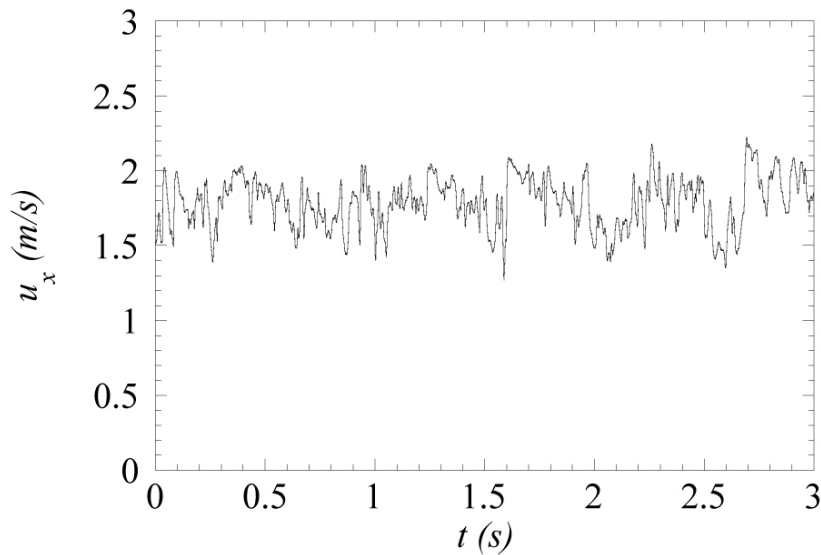
Turbulence de grille



(Comte-Bellot & Corrsin, 1966)

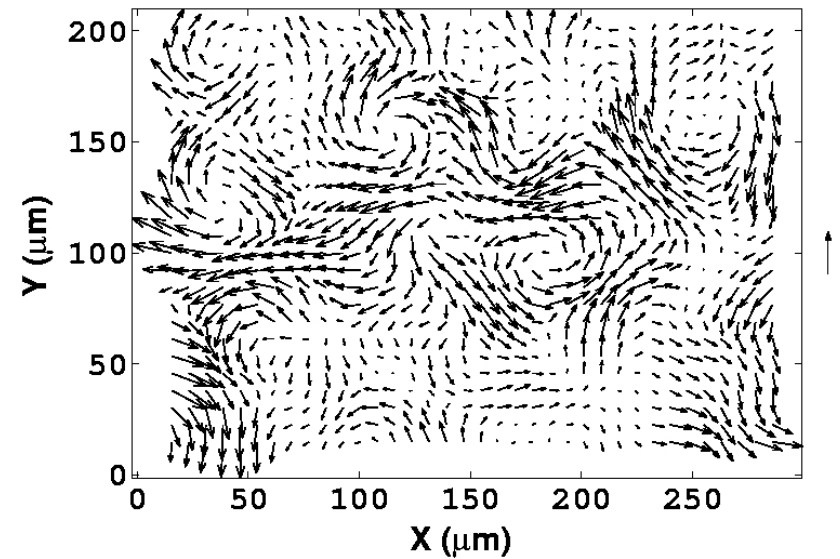
Mesures expérimentales des fluctuations turbulentes

Instantané en 1 point (HWA, LDV)



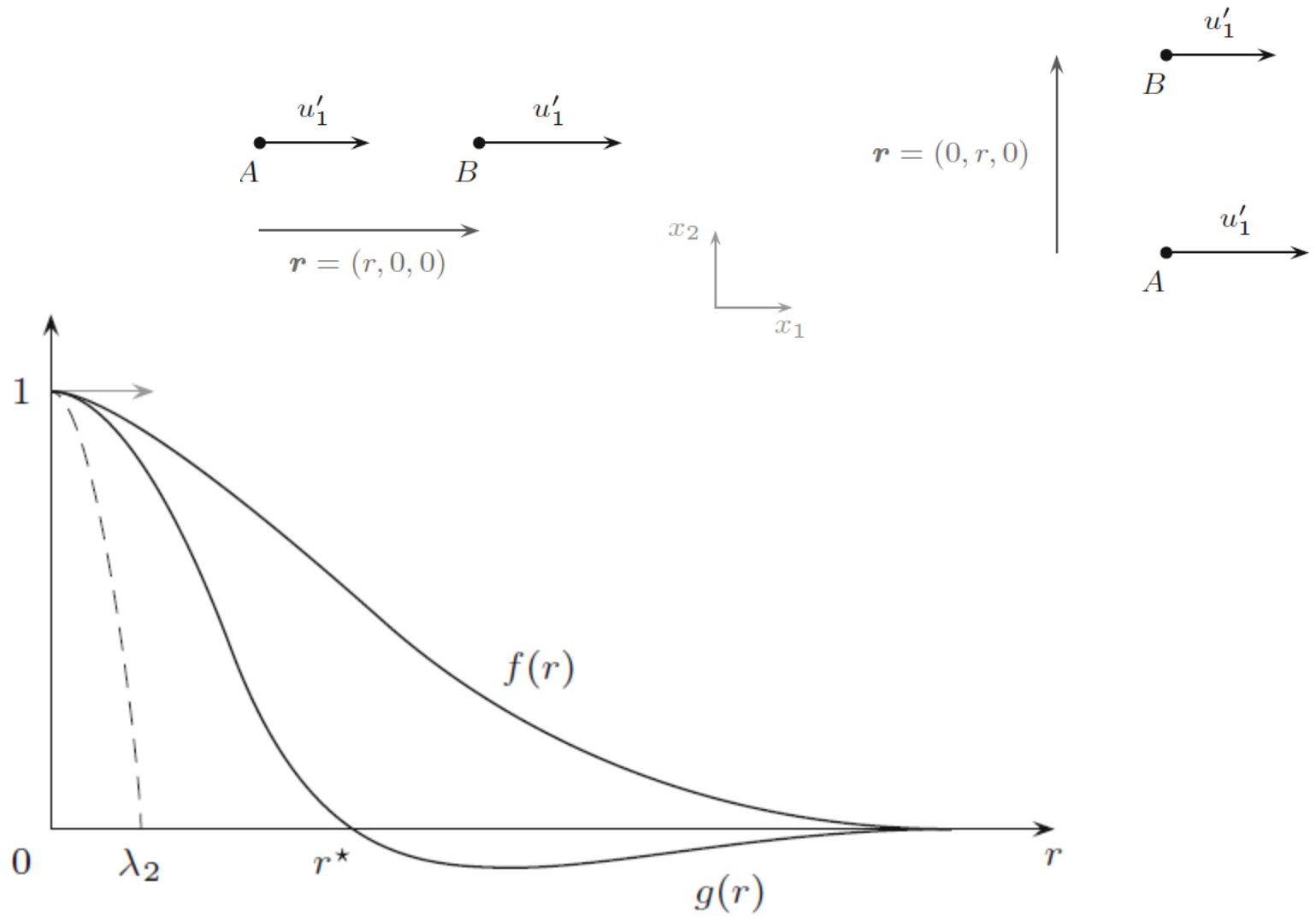
Résolution temporelle: 10 kHz
(Equivalent spatial : $\Delta x = U \Delta t$)

Instantané 2D (PIV)

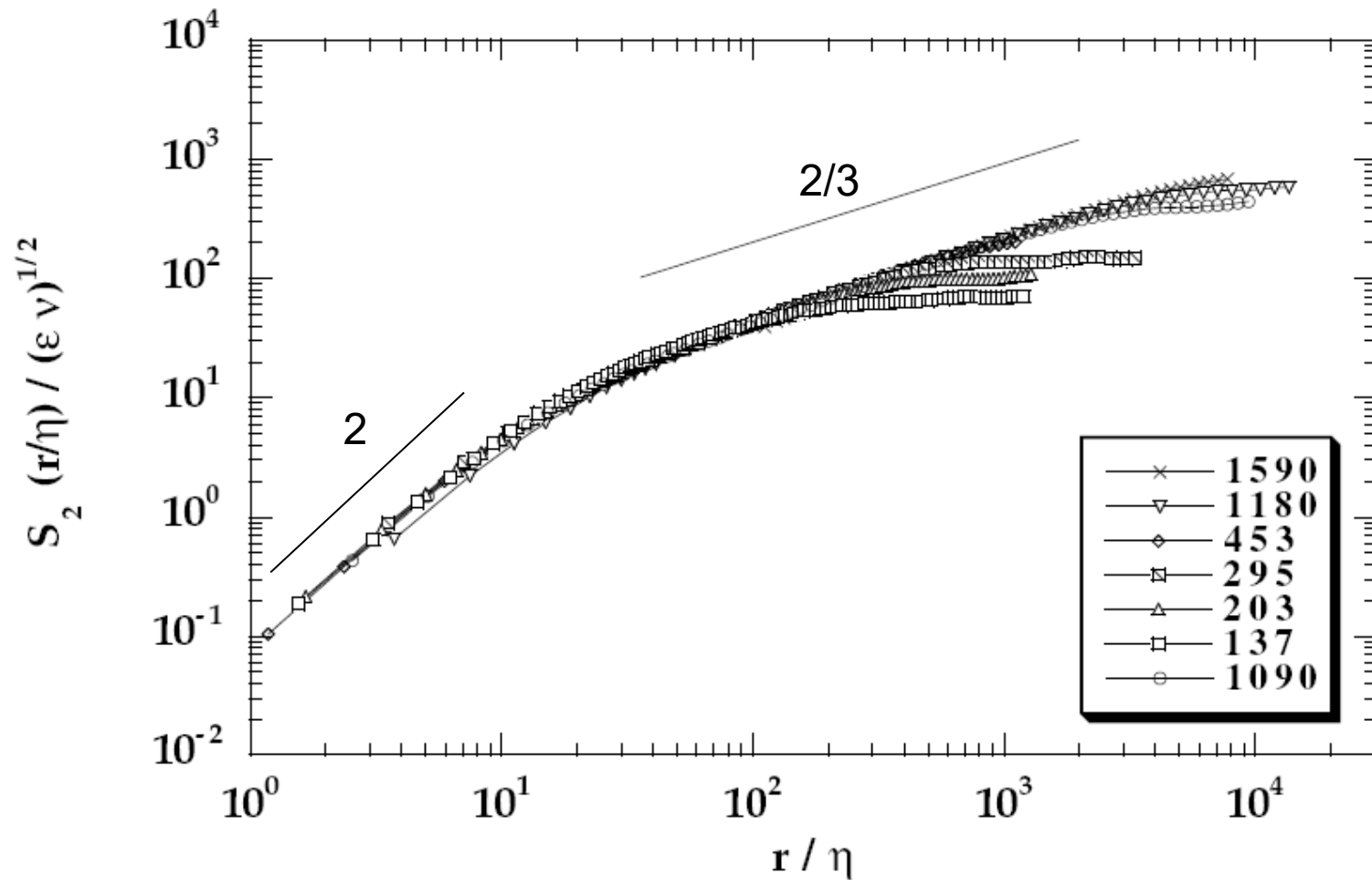


Résolution temporelle: 10Hz – 1kHz
Résolution spatiale : 1 mm

Autocorrélation de vitesse longitudinale et transverse

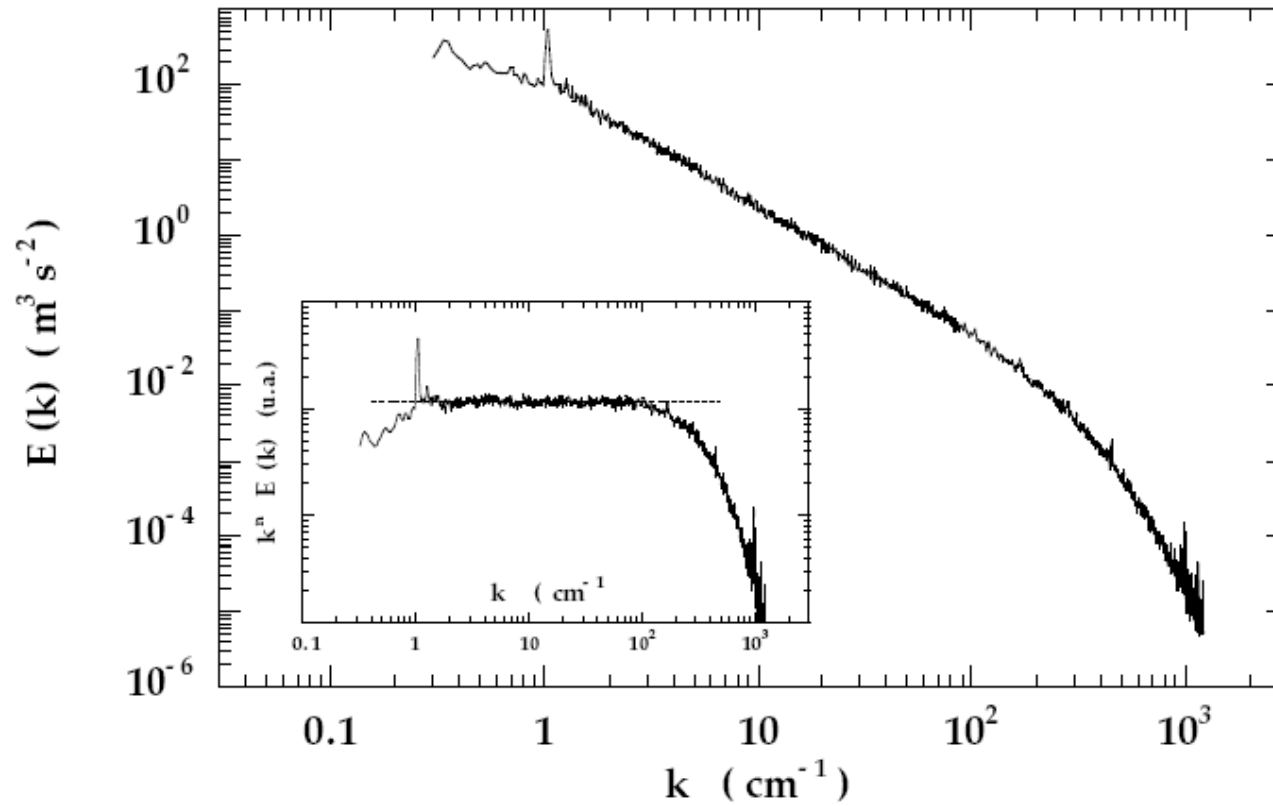


Fonction de structure d'ordre 2 : $D_{LL}(r) = \langle [u_L(x+r) - u_L(x)]^2 \rangle$

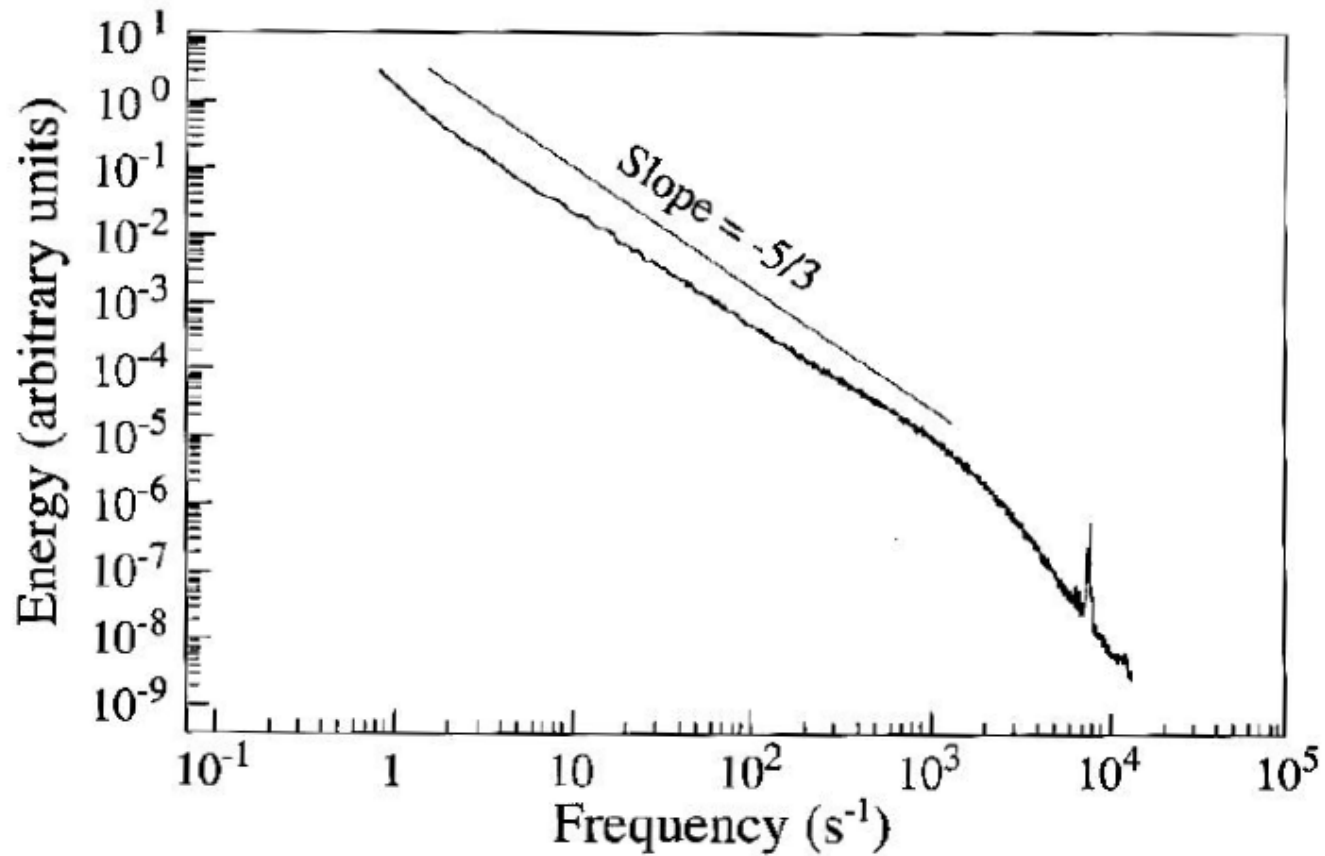


Experiance : turbulence dans l'hélium, T= 5 K

Spectre d'énergie 1D



Spectre d'énergie 1D



Soufflerie S1, Onera

Spectre d'énergie 1D : compilation d'expériences

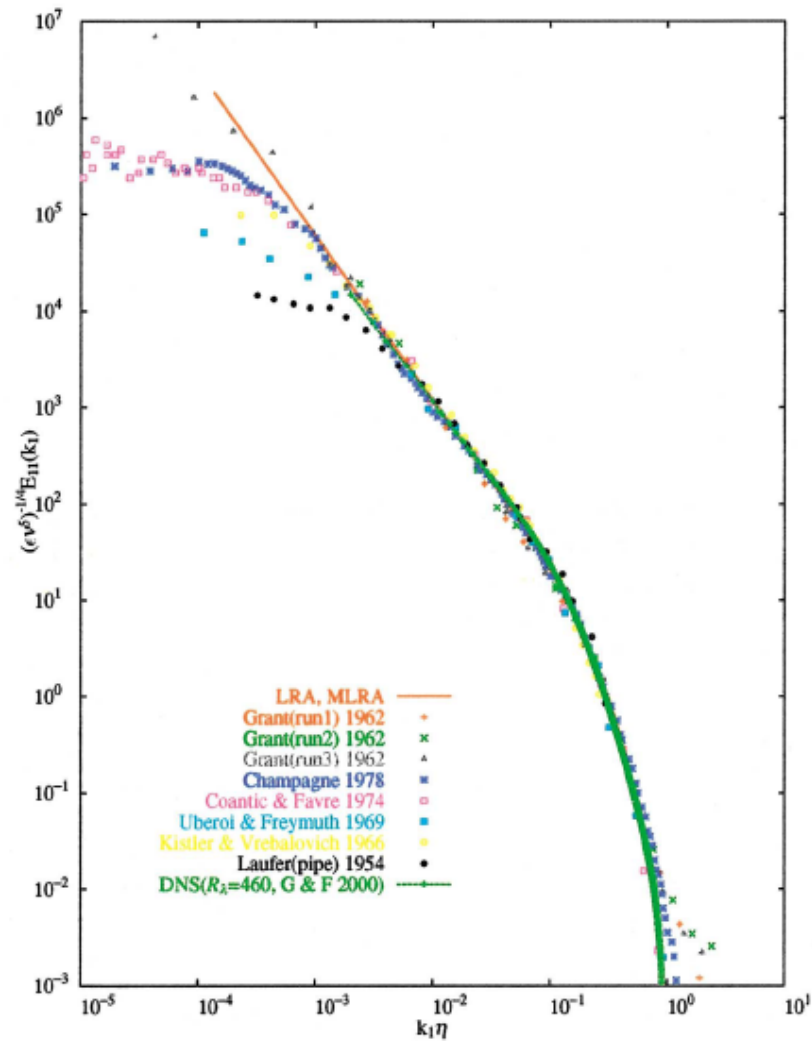
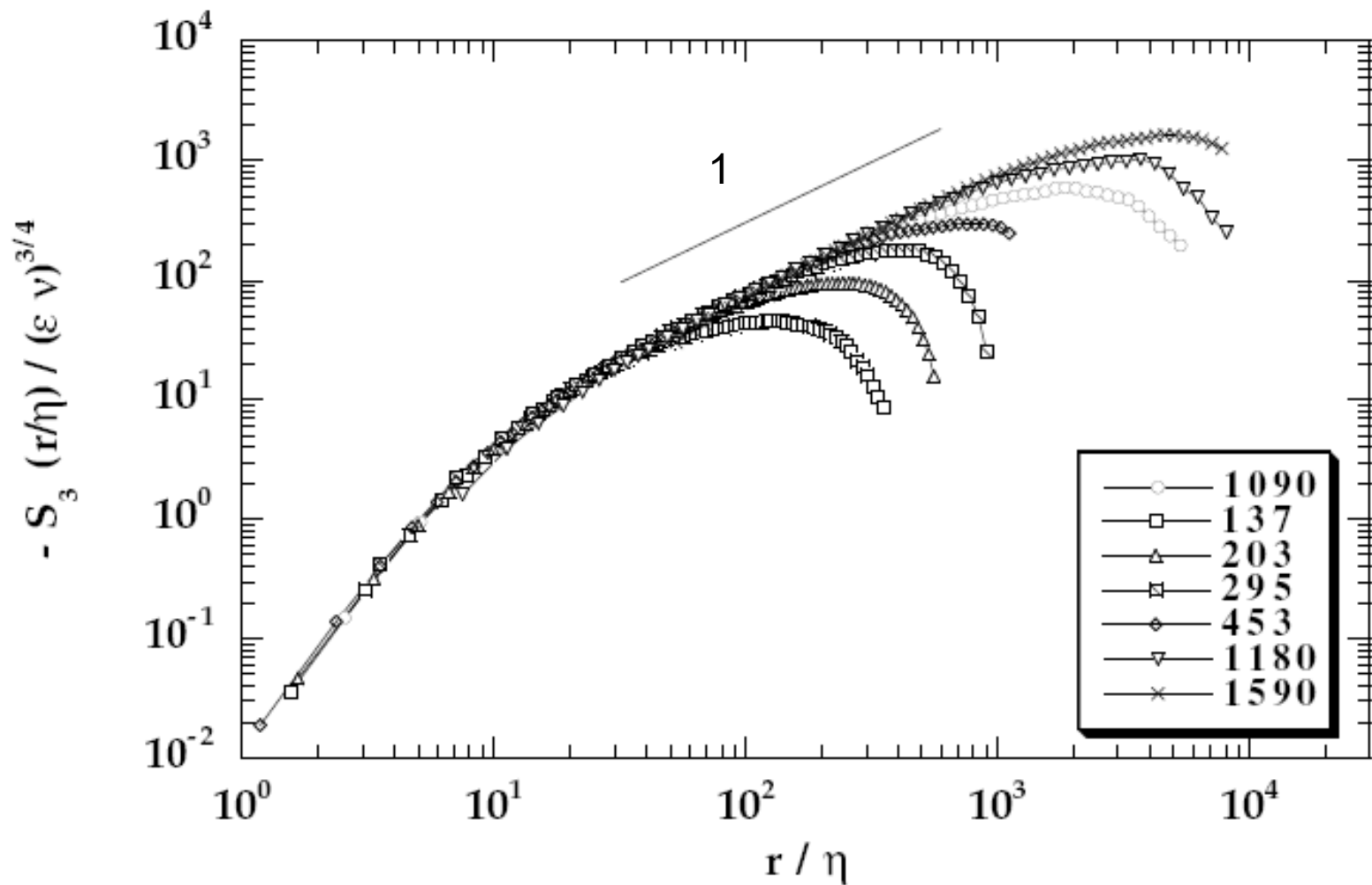
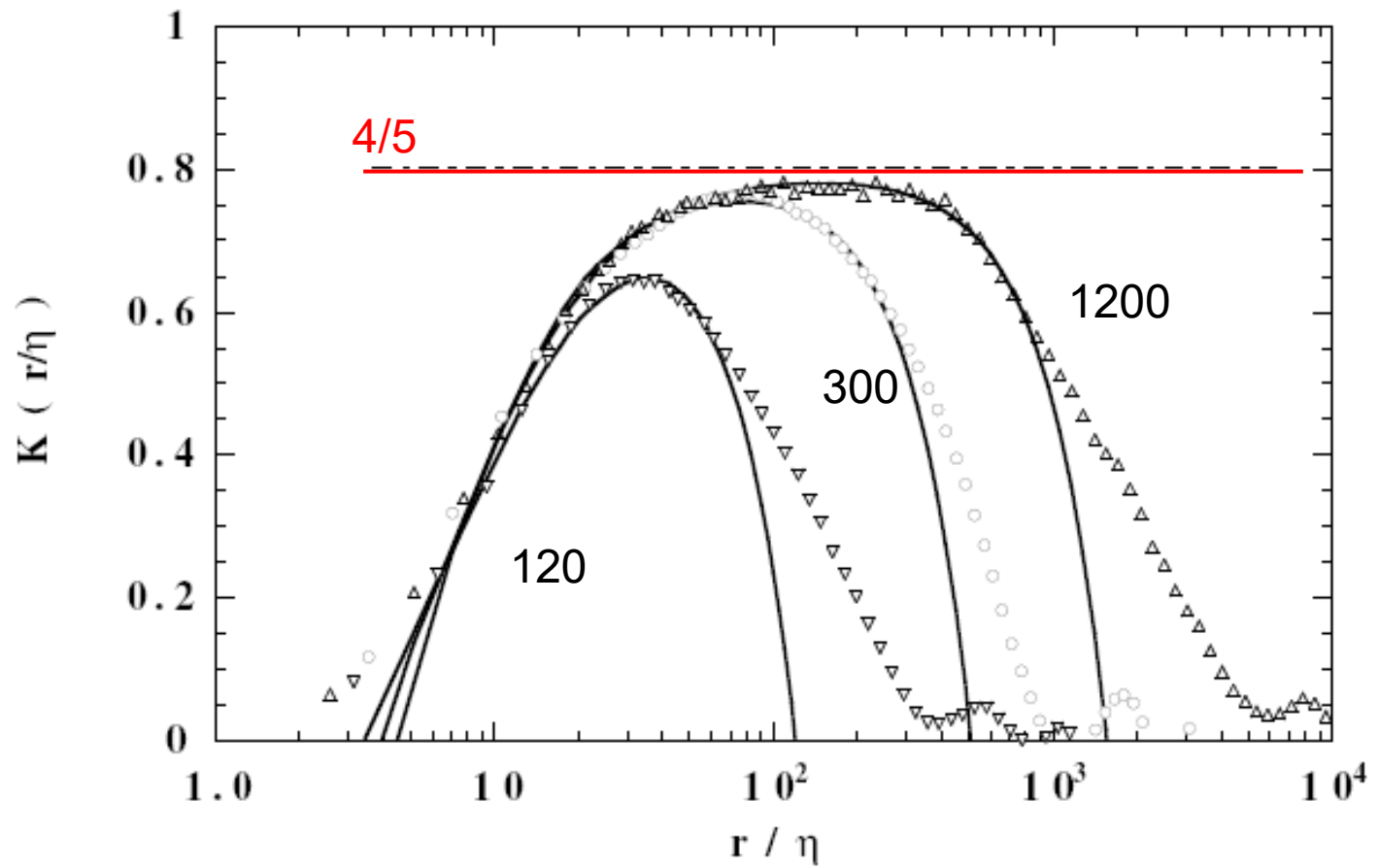


Fig. 1. Comparison of one-dimensional energy spectra. Symbols: experimental, solid red line; Statistical theory (LRA and MLRA), solid green line with plus symbol; present DNS ($R_\lambda = 460$).

Fonction de structure d'ordre 3 : $D_{LLL}(r) = \langle [u_L(x+r) - u_L(x)]^3 \rangle$



Fonction de structure d'ordre 3 adimensionnée : $-D_{LLL}(r) / \varepsilon r$



Asymétrie des distributions des gradients de vitesse du/dx

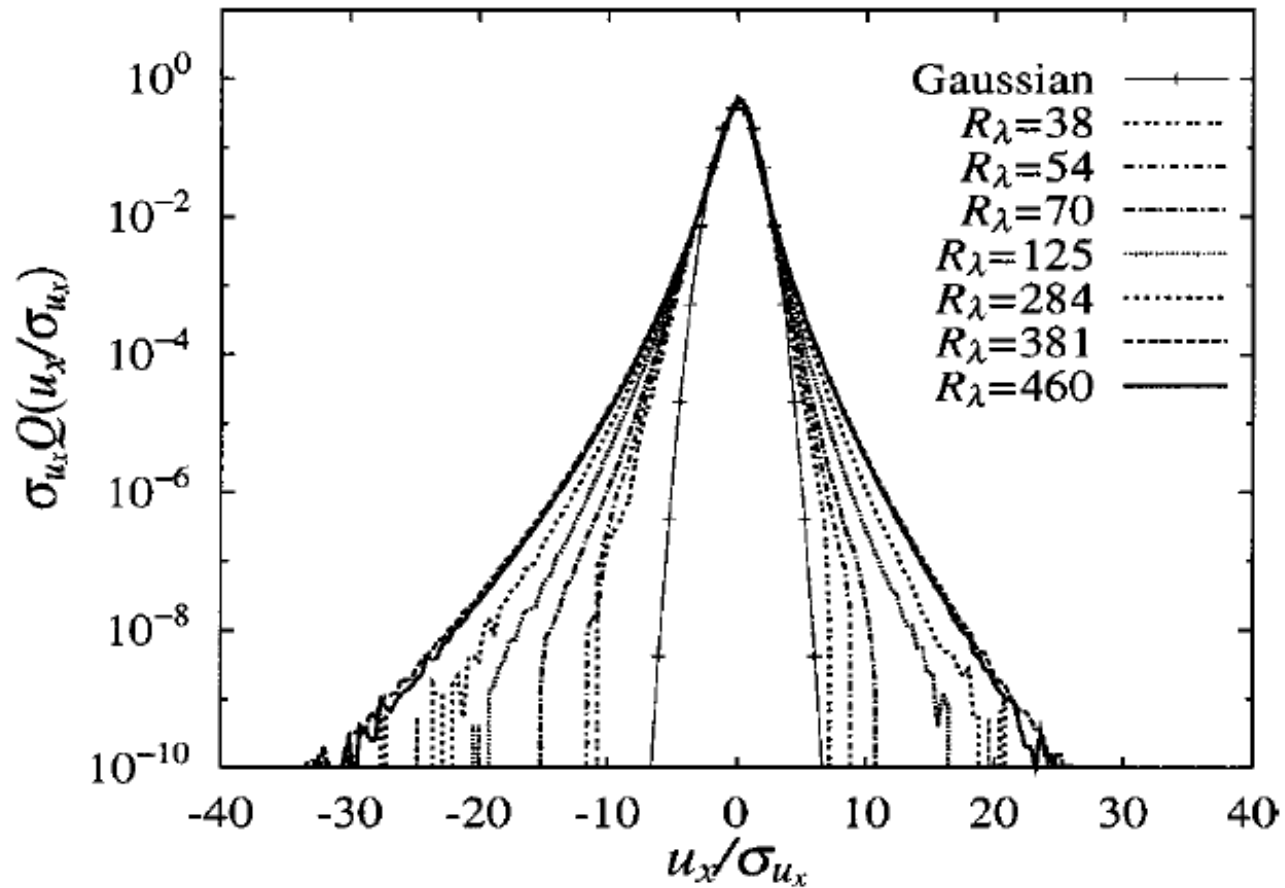


FIG. 6. Variation of the longitudinal velocity derivative PDF with the Reynolds number.

Coefficient d'asymétrie (*skewness*) : $S(r) = - \langle \Delta u_r^3 \rangle / \langle \Delta u_r^2 \rangle^{3/2}$

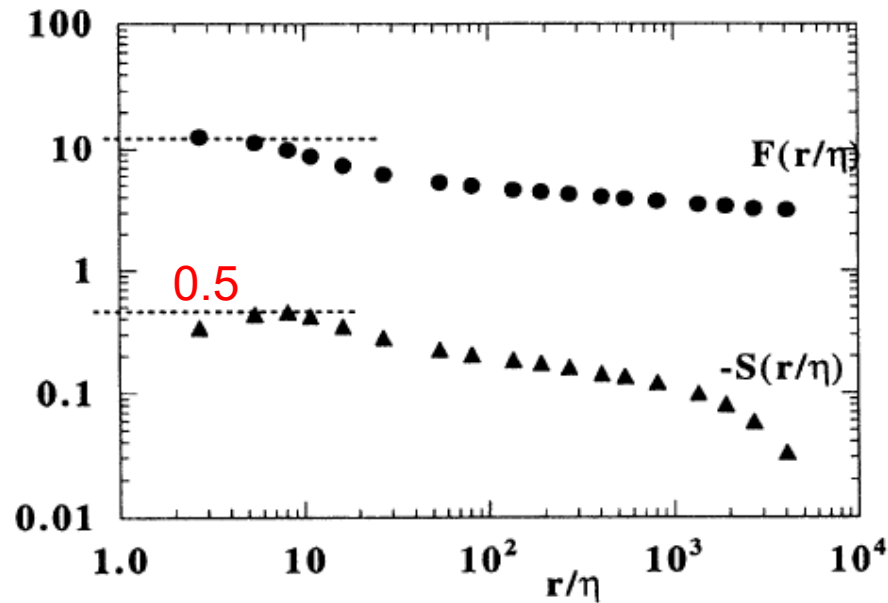
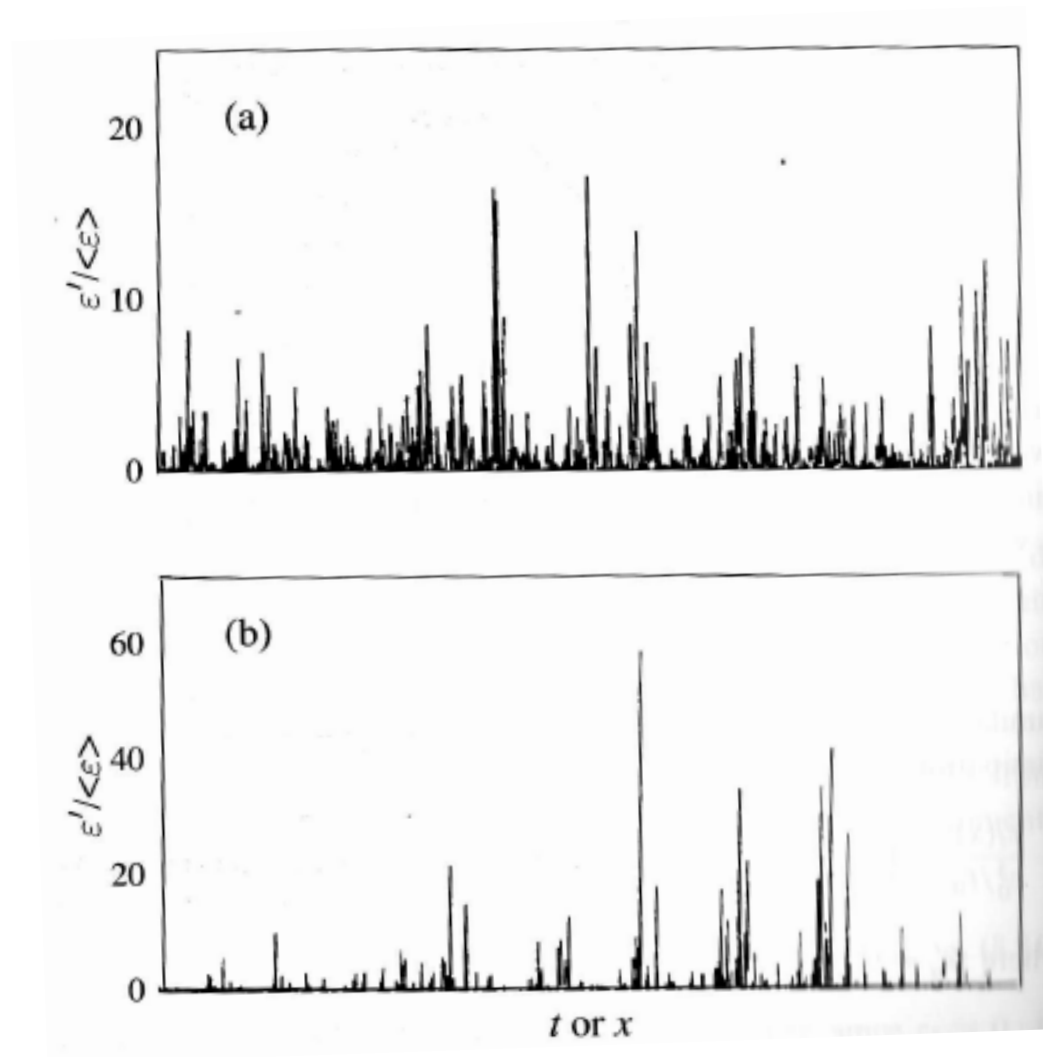


FIG. 11. Evolution of the skewness $-S(r)$ and flatness $F(r)$ of the velocity increments, with the reduced scale r/η , for $R_\lambda \approx 1548$; in this case, $\eta \approx 8.4 \mu\text{m}$ and $l_w/\eta \approx 1$. The broken lines correspond to the values of $-S$ and F which we have used in Fig. 13.

Intermittence de la dissipation $\varepsilon = 15 \nu \langle (du/dx)^2 \rangle$



Intermittence de la dissipation ε :
très grande inhomogénéité spatiale

Energie

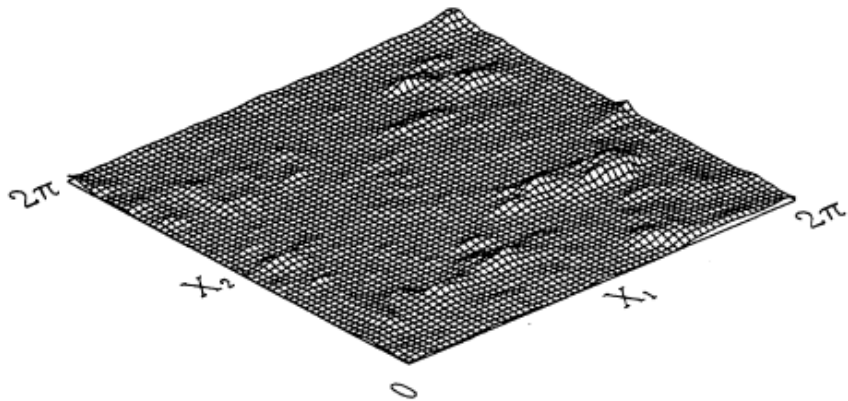


FIG. 11. Bird's-eye view of the u_1^2 field at the $x_3=0$ plane.

Dissipation

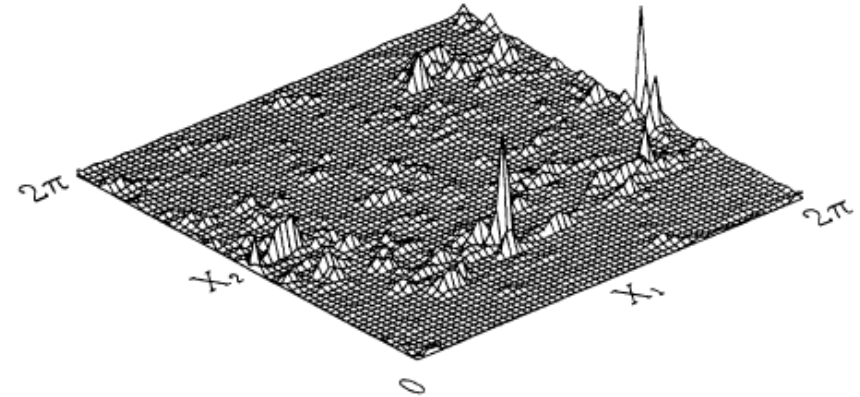
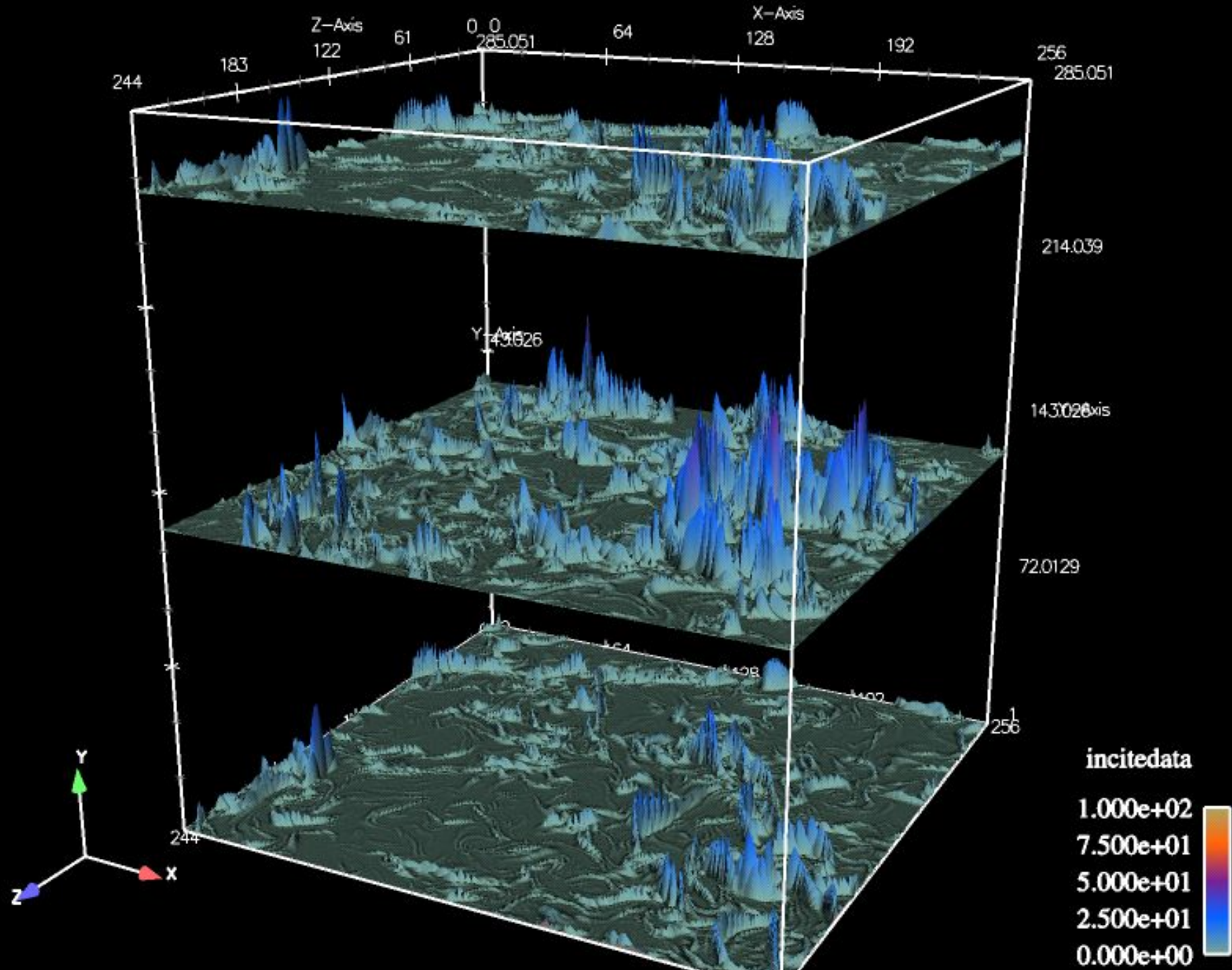
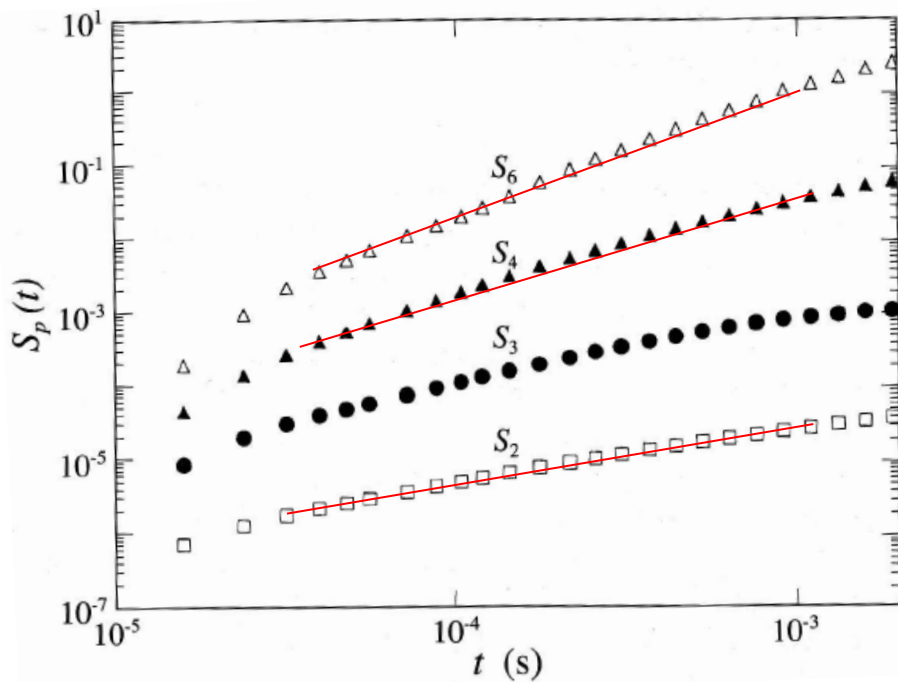


FIG. 13. Bird's-eye view of the $(\partial u_1 / \partial x_2)^2$ field at the same plane as in Fig. 11.



Lois d'échelles anormales :

$$S_p(r) = \langle (u_L(x+r) - u_L(x))^p \rangle \sim r^{\zeta(p)} \quad \text{avec} \quad \zeta(p) \neq p/3$$



Maurer et al, 1994

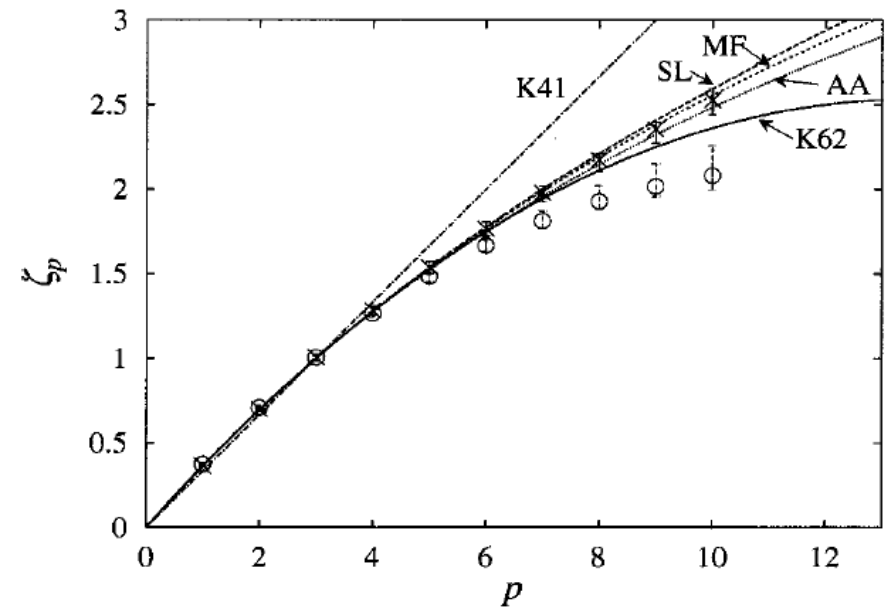
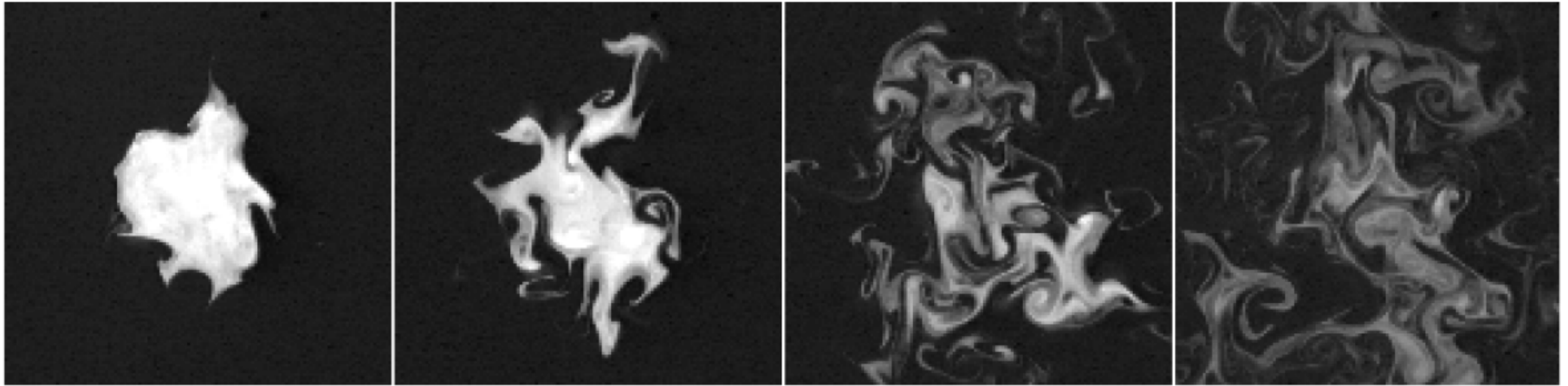


FIG. 32. Variation of the scaling exponents ζ_p^L and ζ_p^T when $R_\lambda = 460$. Symbols are the results of the present DNS, star: ζ_p^L , circle: ζ_p^T . SL, MF, AA, and K62 are the curves by She and Lévéque model (Ref. 70), Yakhot's mean field theory (Ref. 72), Arimitsu and Arimitsu's generalized entropy theory with $\mu=0.25$, and K62 with $\mu=0.25$, respectively.

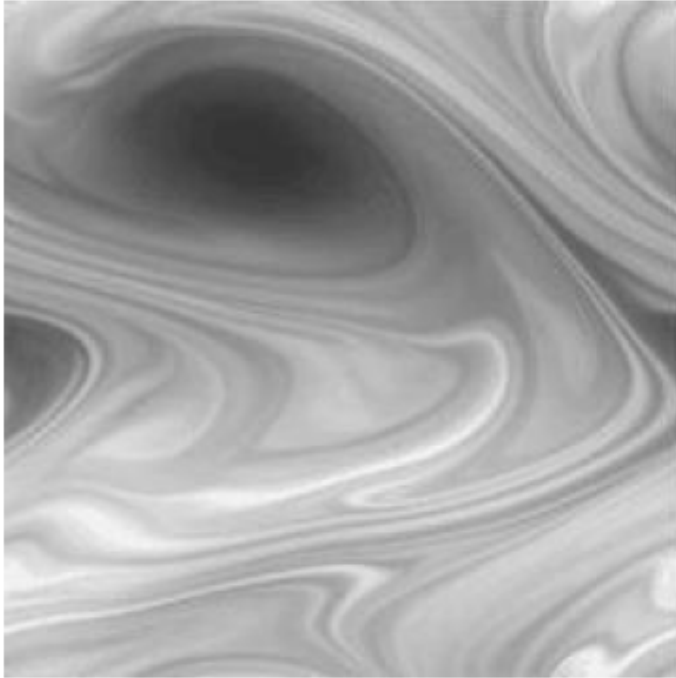


Mélange d'une tache de colorant en turbulence 2D

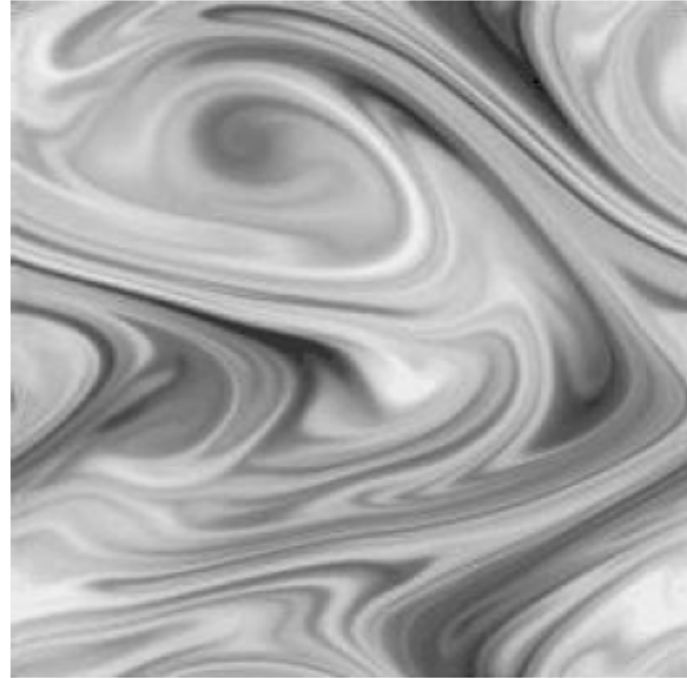


Jullien et Tabeling (2001)

Mélange 2D



vorticité

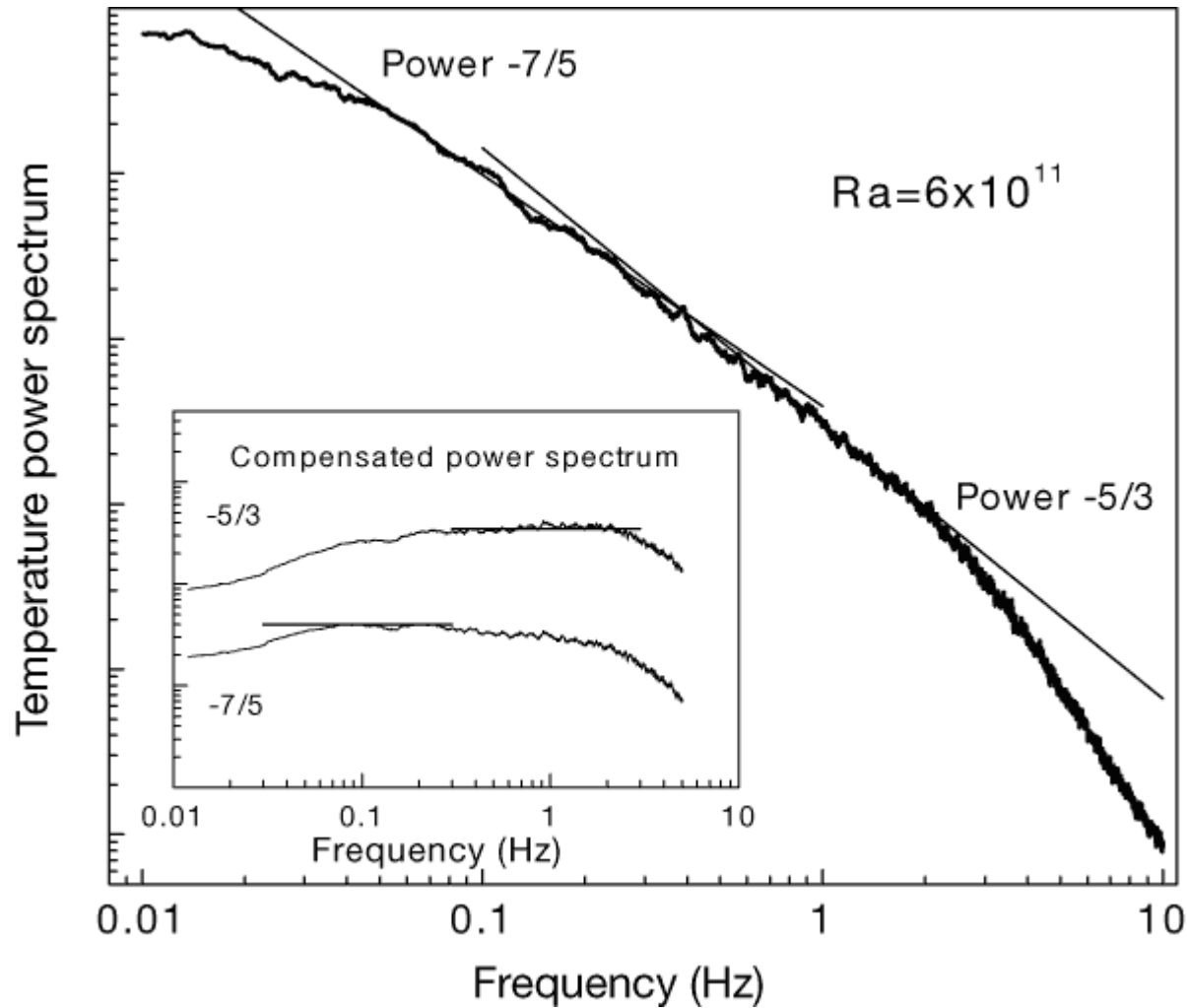


scalaire

Mélange turbulent : cas où $Sc \gg 1$



Spectre de température (scalaire passif ?)



[Turbulent convection at very high Rayleigh numbers](#)

J. J. Niemela, L. Skrbek, K. R. Sreenivasan and R. J. Donnelly

Nature 404, 837-840 (2000)