Corrigé du TD 3 - exercice 4.

Equation pour la variance d'un champ scalaire.

1°) $$\frac{\partial}{\partial t}(\bar\theta+\theta') + (\bar u_j + u_j')\frac{\partial}{\partial x_j}(\bar\theta+\theta') = K\frac{\partial^2}{\partial x_j^2}(\bar\theta+\theta'). \quad (1)$$

on développe et on moyenne :

$$\frac{\partial}{\partial t}\bar\theta + \bar u_j\frac{\partial}{\partial x_j}\bar\theta = -\frac{\partial}{\partial x_j}(\overline{u_j'\theta'}) + K\frac{\partial^2\bar\theta}{\partial x_j^2}. \quad (2)$$

2°) On calcule (2)−(1) :

$$\underbrace{\frac{\partial}{\partial t}\theta' + \bar u_j\frac{\partial}{\partial x_j}\theta'}_{\frac{D}{Dt}\theta'} + u_j'\frac{\partial}{\partial x_j}\theta' + u_j'\frac{\partial\bar\theta}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\overline{u_j'\theta'}\right) + K\frac{\partial^2\theta'}{\partial x_j^2} \quad (3).$$

3°) $\overline{\theta'(3)} \Rightarrow$

$$\frac{D}{Dt}\left(\frac{1}{2}\overline{\theta'^2}\right) = \overline{\theta'\frac{\partial}{\partial x_j}\Big(\underset{(a)}{-u_j'\bar\theta} + \underset{(b)}{\overline{u_j'\theta'}} \underset{(c)}{-u_j'\theta'}\Big)} + \underset{(d)}{K\overline{\theta'\frac{\partial^2\theta'}{\partial x_j^2}}}$$

$$(a) = -\overline{\theta'\frac{\partial}{\partial x_j}(u_j'\bar\theta)} = -\overline{\theta' u_j'}\frac{\partial\bar\theta}{\partial x_j} - \bar\theta\,\overline{\theta'\underbrace{\frac{\partial u_j'}{\partial x_j}}_{=0}}.$$

$$(b) = \underbrace{\overline{\theta'}}_{=0}\frac{\partial}{\partial x_j}(\overline{u_j'\theta'}) = 0$$

$$(c) = -\overline{\theta'\frac{\partial}{\partial x_j}(u_j'\theta')} = -\frac{\partial}{\partial x_j}(\overline{u_j'\theta'^2}) + \underbrace{\overline{u_j'\theta'\frac{\partial\theta'}{\partial x_j}}}_{\overline{\theta'\frac{\partial}{\partial x_j}(u_j'\theta')} - \overline{\theta'^2\frac{\partial u_j'}{\partial x_j}}}$$

d'où $$(c) = -\frac{\partial}{\partial x_j}(\overline{u_j'\theta'^2}) + (c)$$

soit $$(c) = -\frac{\partial}{\partial x_j}\left(\frac{1}{2}\overline{u_j'\theta'^2}\right).$$

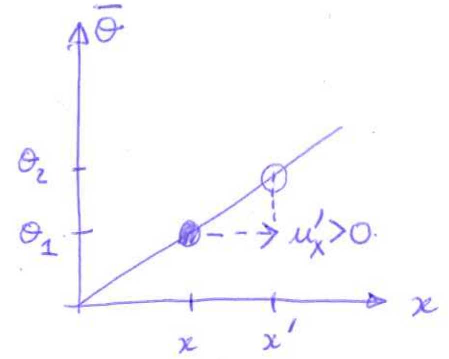
$$(d) = \kappa\, \overline{\theta' \frac{\partial}{\partial x_j}\left(\frac{\partial \theta'}{\partial x_j}\right)} = \kappa \frac{\partial}{\partial x_j}\left(\overline{\theta' \frac{\partial \theta'}{\partial x_j}}\right) - \kappa\, \overline{\left(\frac{\partial \theta'}{\partial x_j}\right)^2}$$

D'où $\quad (a) = P_\theta \quad , \quad (b) = 0.$

$$(c) + (d) = \frac{\partial}{\partial x_j}\left(\frac{1}{2}\overline{u_j'\theta'^2} + \kappa \frac{\partial}{\partial x_j}\left(\frac{1}{2}\overline{\theta'^2}\right)\right) - \kappa\, \overline{\left(\frac{\partial \theta'}{\partial x_j}\right)^2}$$

$$= T_\theta - \varepsilon_\theta .$$

4°)



Supposons une fluctuation $u_x'>0$. une particule fluide en x de température $\overline{\Theta} = \Theta_1$ arrive en x' dans une région de température $\overline{\Theta}(x') = \Theta_2 > \Theta_1$. Cela induit donc en x' une fluctuation $\theta' = \Theta_1 - \Theta_2 < 0$

D'où $\overline{u_x'\theta'} < 0$

Donc $P_\theta > 0 \Rightarrow$ Terme de production.

5°) Il faut modéliser le terme T_θ.
Seul le terme de flux turbulent de variance $\frac{1}{2}\overline{u_j'\theta'^2}$ est inconnu. On peut introduire une diffusivité turbulente K_θ telle que $\frac{1}{2}\overline{u_j'\theta'^2} = K_\theta \frac{\partial k_\theta}{\partial x_j}$.

on a donc $\quad T_\theta = \frac{\partial}{\partial x_j}\left[(\kappa + K_\theta)\frac{\partial k_\theta}{\partial x_j}\right]$

d'où le modèle :

$$\begin{cases} \dfrac{D}{Dt} k_\theta = \dfrac{\partial}{\partial x_j}\left[(\kappa + K_\theta)\dfrac{\partial k_\theta}{\partial x_j}\right] + P_\theta - \varepsilon_\theta . \\[2ex] \dfrac{D}{Dt} \varepsilon_\theta = \dfrac{\partial}{\partial x_j}\left[(\kappa + K_\theta')\dfrac{\partial \varepsilon_\theta}{\partial x_j}\right] + C_{1\theta}\dfrac{P_\theta \varepsilon_\theta}{k_\theta} - C_{2\theta}\dfrac{\varepsilon_\theta^2}{k_\theta} \end{cases}$$