TD5. ex 1.

1°) $u_r' \approx (\varepsilon r)^{1/3}$ $\qquad \eta \ll r \ll L$

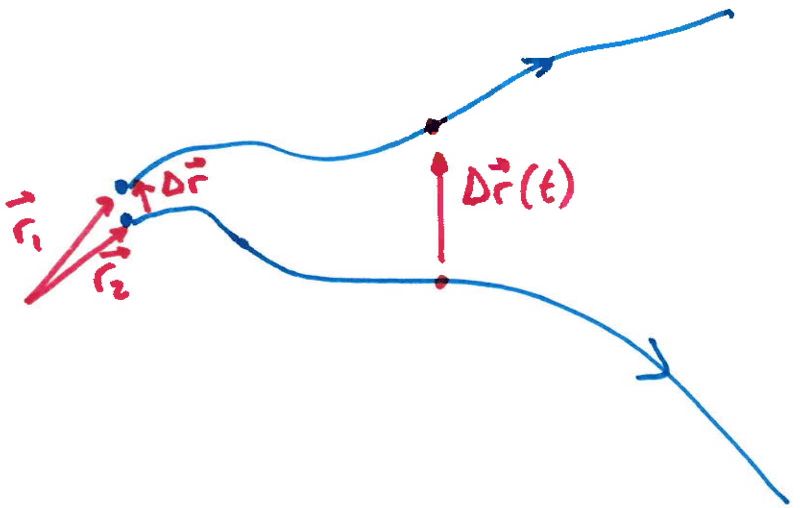
2°) $r \approx L$, $\varepsilon \approx \frac{u'^3}{L}$ $\qquad u' = 0.3 \times 200$ km/h. $\simeq 17$ m/s.

$L = 1000$ km. (?)

$\hookrightarrow \varepsilon \simeq 0{,}005 \ m^2/s^3$ (W/Kg).

Ech. Kolmogorov : $\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \simeq 6$ mm.

3°)



$n = 1$ ou 2

$$\boxed{\frac{d}{dt}\vec{r}_n = \vec{v}(\vec{x} = \vec{r}_n(t), t)}$$

Lagrangienne $\qquad$ Eulerienne

$$\vec{r}_n = \vec{r}_n(0) + \int_0^t \vec{v}(\vec{r}_n, t')\,dt'$$

$$\Delta\vec{r} = \vec{r}_2(t) - \vec{r}_1(t).$$

$$\frac{d}{dt}\overline{\Delta\vec{r}} = \overline{\vec{v}(\vec{x}=\vec{r}_2(t),t)} - \overline{\vec{v}(\vec{x}=\vec{r}_1(t),t)}.$$

$$\overline{\Delta\vec{r}} = \vec{0} \quad \text{MAIS} \quad \overline{(\vec{\Delta r})^2} \neq 0.$$

4°) $$\frac{1}{2}\frac{d}{dt}\overline{\Delta r^2} = \overline{\Delta\vec{r}\cdot\frac{d}{dt}\Delta\vec{r}} = \overline{\Delta\vec{r}\cdot[\vec{v}(\vec{r}_2)-\vec{v}(\vec{r}_1)]} \neq 0.$$

5°) $$\overline{\Delta\vec{r}\cdot\Delta\vec{v}} = \alpha R' u'_R \qquad \text{où} \quad R'(t) = \sqrt{\overline{\Delta r^2}}$$

$$u'_R \approx (\varepsilon R')^{1/3}$$

d'où $$\underbrace{\frac{1}{2}\frac{d}{dt}R'^2}_{R'\frac{d}{dt}R'} = \alpha R'(\varepsilon R')^{1/3}$$

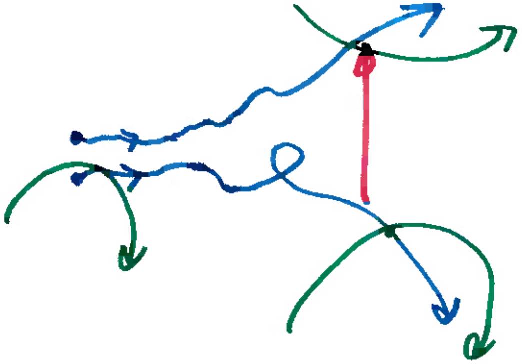
$$\int_{R'(0)}^{R'(t)} \frac{dR'}{R'^{1/3}} = \alpha\varepsilon^{1/3}\int_0^t dt$$

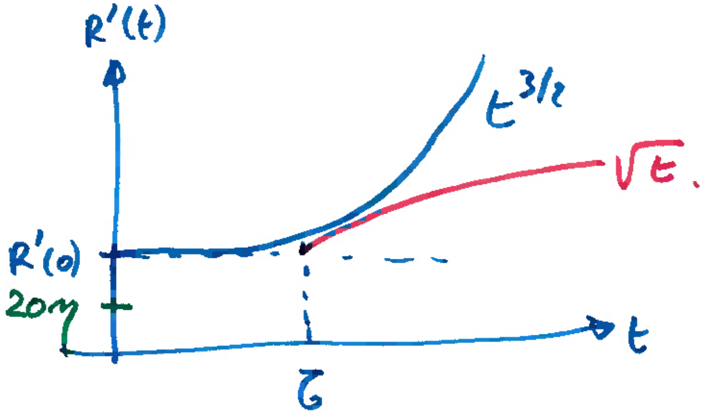
$$\left[\frac{R'^{2/3}}{2/3}\right]_{R'(0)}^{R'(t)} = \alpha\varepsilon^{1/3} t$$

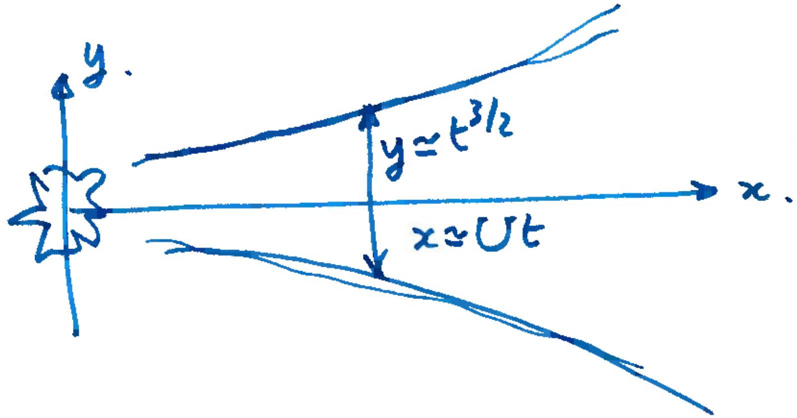
$$R'^{2/3}(t) = R'(0)^{2/3} + \frac{2}{3}\alpha\varepsilon^{1/3} t$$

$$R'(t) = R'(0)\left[1 + \frac{2}{3}\frac{\alpha\varepsilon^{1/3}}{R'(0)^{2/3}}t\right]^{3/2}$$

$$\boxed{R'(t) = R'(0)\left[1 + t/\tau\right]^{3/2}}$$





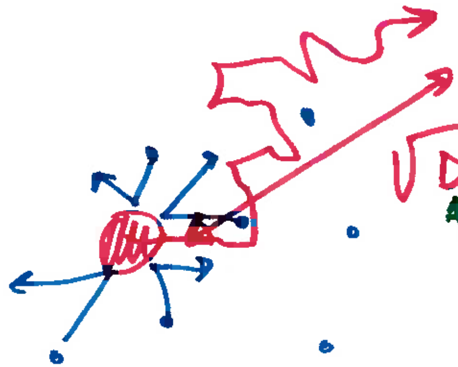


Dispersion Turbulente.

↳ $t^{3/2}$

Mouvement Brownien.

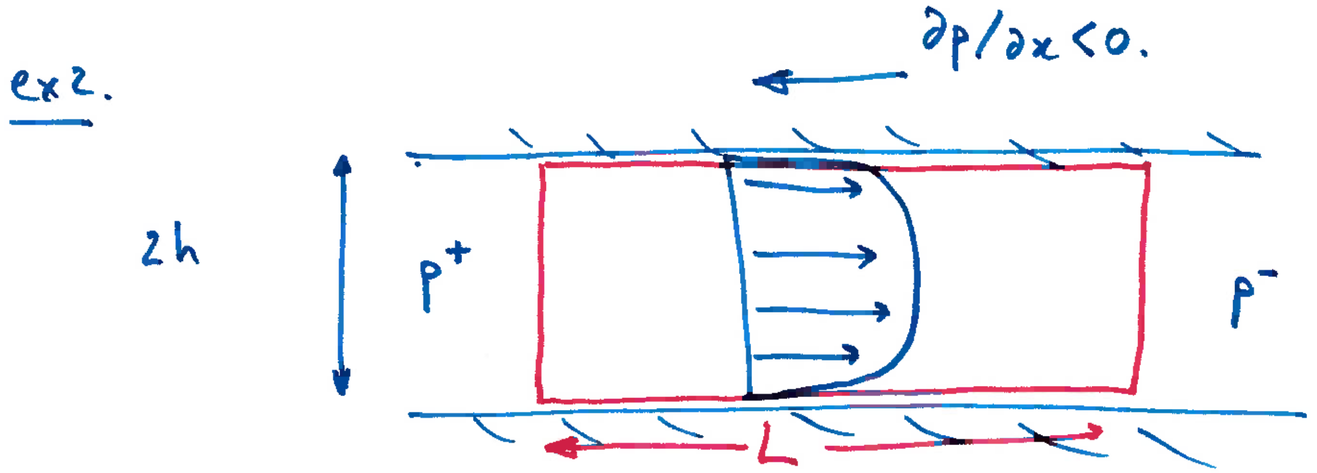
$t^{1/2}$



$\sqrt{Dt} = x(t)$

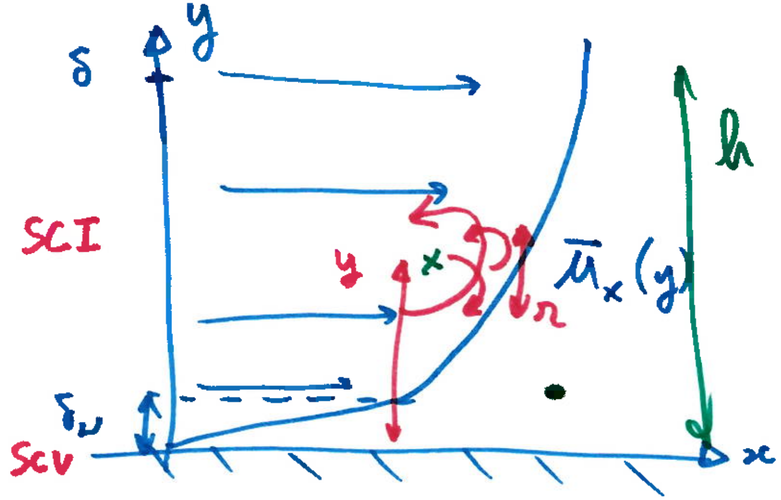
$D \propto t^2$

ex2.



$\delta_\nu = \dfrac{\nu}{u^*}$ où u^* vitesse de friction telle que $\tau_0 = \rho u^{*2}$

avec $\boxed{\tau_0 = \Delta p \, \dfrac{2h}{L}}$



$p_+(2hb) - p_-(2hb) = \tau_0 \, 2Lb$

↳

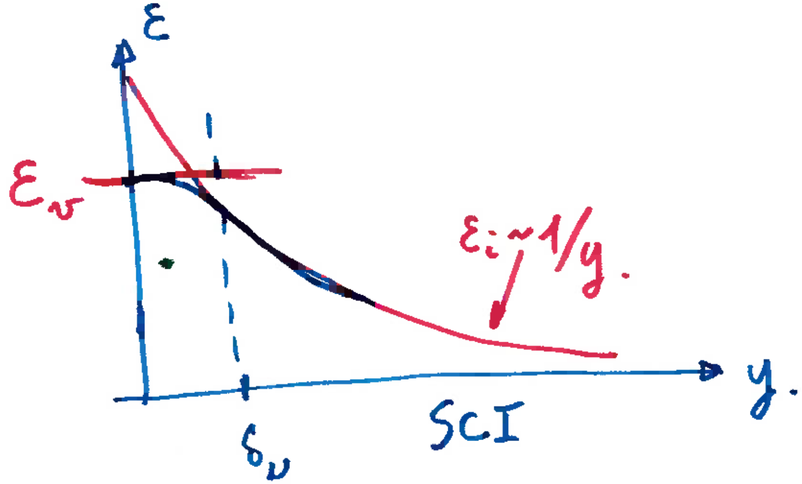
$$\begin{cases} \overline{u}_x(y) = u^* \dfrac{y}{\delta_\nu} & (\mathrm{SCV}) \\ \overline{u}_x(y) = u^* \left(\dfrac{1}{k} \ln\left(\dfrac{y}{\delta_\nu}\right) + A \right) & (\mathrm{SCI}). \end{cases}$$

$$\varepsilon_i = -\overline{u_i' u_j'} \frac{\partial \overline{u_i}}{\partial x_j} = \varepsilon_d(y) = \nu \overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2}$$

$$(18) \Rightarrow \varepsilon_i^{(y)} = -\overline{u_x' u_y'} \frac{\partial \overline{u_x}}{\partial y} \simeq u^{*2} \times \frac{u^*}{\kappa y} = \boxed{\frac{u^{*3}}{\kappa y}}$$

$y \searrow \Rightarrow \varepsilon_i \nearrow$ $\qquad$ ε_i maximum en $y \simeq \delta_\nu$.

2°) dans la SCV, $\quad \varepsilon_\nu = \nu \left(\frac{\partial \overline{u_x}}{\partial y}\right)^2 = \nu \left(\frac{u^*}{\delta_\nu}\right)^2 = \nu \frac{u^{*2}}{(\nu/u^*)^2} = \boxed{\frac{u^{*4}}{\nu}}$ $\quad \simeq$



$$\varepsilon_i(y = \delta_\nu) = \frac{u^{*3}}{\kappa \nu / u^*}$$
$$= \frac{1}{\kappa} \frac{u^{*4}}{\nu} = \frac{\varepsilon_i}{\kappa}$$

3°) en y. $\qquad v'_r(y) \simeq \big(\underbrace{\varepsilon_i(y)}_{\frac{u^{*3}}{\kappa y}}\, r\big)^{1/3}$ avec $\eta(y) \ll r \ll y$.

$$v'_r = \left(\frac{u^{*3}}{\kappa y}\, r\right)^{1/3} \simeq \frac{u^*}{\kappa^{1/3}} \left(\frac{r}{y}\right)^{1/3} \ll u^*$$

4°) <u>Ech. de Kolmogorov.</u>

$$\eta(y) = \left(\frac{\nu^3}{\varepsilon(y)}\right)^{1/4} = \left(\frac{\nu^3}{u^{*3}/y}\right)^{1/4} = \left(\frac{\nu^3}{u^{*3}h^3}\,\frac{y h^3}{\ }\right)^{1/4}$$

$$\boxed{\eta(y) = \underbrace{Re^{-3/4}\, h}_{\eta(y=h)} \underbrace{\left(\frac{y}{h}\right)^{1/4}}_{<1.}} \qquad Re = \frac{h u^*}{\nu}.$$

$$\eta(y=\delta_\nu) = Re^{-3/4}\, h \underbrace{\left(\frac{\nu}{u^* h}\right)^{1/4}}_{Re^{-1/4}} = h\, Re^{-1} = h . \frac{\nu}{u^* h} = \underline{\delta_\nu} .$$